1696


The
Elements
of
EUCLID
Explained
by the Famous
Monr. De Chales

62501

THE ELEMENTS OF EUCLID Explain'd,

in a *New*, but most *Easie* method:

Together with
The Use of every Proposition through all parts of the Mathematicks.

Written in *French*, by that Excellent Mathematician,
F. CLAUD. FRANCIS MILLIET de CHALES,
of the *Society of JESUS*.

Now made *English*, and a multitude of Errors Corrected, which had escap'd in the Original.

The Second Edition.

LONDON,
Printed for *M. Gillyflower* at the *Spread Eagle* in *Westminster-Hall*, and *W. Freeman* at the *Bible* over against the *Middle Temple Gate*, in *Fleet-street*. 1696.

THE PREFACE.

HAving for a long time obſerv'd, that moſt of thoſe that take in hand the Elements *of* Euclid, *are apt to diſlike them, becauſe they cannot preſently diſcern, to what end thoſe ſeemingly inconſiderable, and yet difficult Propoſitions, can conduce: I thought I ſhould do an acceptable piece of ſervice, in not only rendring them as eaſie as poſſible, but alſo adding to each Propoſition a brief account of ſome Uſe, that is made of them in the other parts of the* Mathematicks. *In proſecuting which deſign, I have been oblig'd to change ſome Demonſtrations, that ſeem'd too intricate and perplex'd, and above the ordinary capacity of Beginners, and to ſubſtitute others more intelligible in their ſtead. For the ſame reaſon, I have demonſtrated the fifth Book after a method, much more clear, than that by* Equimultiples, *formerly uſed. I would not be thought to have ſet down all the Uſes, that may be*

made of these Propositions: to have done that, would have oblig'd me to have compris'd the whole Mathematicks *in this one Book; which would have render'd it both too large, and too difficult. But I have contented my self with the choice of such, as may serve to point out some of the Advantages they afford us, and are also in themselves most clear, and most easie to be apprehended. I have distinguish'd 'em by* * Inverted Commas, *that the Reader may know 'em; not desiring he should dwell too long upon 'em, or labour to understand 'em perfectly at first, since they depend on the Principles of the other Parts.*

This therefore being the design of this small Treatise, I voluntarily offer it to the publick, in an Age, whose Genius *seems more addicted to the* Mathematicks, *than any that has preceded it.*

* Instead of the Authors Italick Character.

Eight

Eight Books of the Elements of EUCLID, *together with the Use of the* Propositions.

THE FIRST BOOK.

'THE design of *EUCLID* in this Book
' is to lay down the First Principles of
' *Geometry*; and to do it methodically,
' he begins with *Definitions*, and the expli-
' cation of the most ordinary Terms. To these
' he adds some *Postulata*; and then Propo-
' sing those known *Maxims*, in which natu-
' ral reason does instruct us, he pretends, not to
' advance a step farther without a *Demonstration*,
' but to convince every man, even the most ob-
' stinate, that will grant nothing, but what is ex-
' torted from him. In the first Propositions he
' treats of *Lines*, and the different *Angles*, which

'are form'd by their concourse; and having oc-
'casion to compare divers *Triangles* together,
'in order to demonstrate the Properties of
'*Angles*, he makes that the business of the
'Eight first Propositions. Then follow some
'Practical Instructions, how to divide an *Angle*
'and a *Line* into two equal parts, and to draw a
'*Perpendicular*. Next he shews the properties
'of a *Triangle*, together with those of *Parallel*
'*Lines*; and having thus finish'd the Explica-
'tion of this first figure, he passes on to *Paral-*
'*lelograms*, teaching the manner of reducing
'any *Polygone*, or multangular figure into one
'more regular. Lastly, he finishes the first
'Book with that famous Proposition of *Pytha-*
'*goras*, That *in every rectangular Triangle the*
'*Square of the* * *Base is equal to the Squares of*
'*both the other sides.*

* He calls that the *Base*, which is commonly call'd the *Hypotenuse*, i. e. the Line that is opposite to the right Angle.

DEFINITIONS.

1. A *Point is that which hath no parts.*

'This Definition must be understood in
'this sense: That quantity, which we conceive
'without distinguishing its parts, or so much as
'considering whether or no it has any, is a *Ma-*
'*thematical* point; which is therefore very
'dif-

'different from those of *Zeno*, which were sup-
'pos'd to be absolutely indivisible, and there-
'fore such, that we may reasonably doubt whe-
'ther they are possible; but the former we can-
'not doubt of, if we conceive them aright.

2. *A Line is length without breadth.*

'The sense of this Definition is the same with the former That quantity, which we conceive as length, without reflecting on its breadth or thickness, is that, which we understand by a Line; though it be impossible to draw a real Line, which will not be of a certain breadth. 'Tis commonly said, that a Line is produc'd by the motion of a Point; which ought to be carefully observ'd; for motion may on that manner produce any quantity whatsoever: But here, we must imagine a Point to be only so mov'd, as to leave one trace in the space, through which it passes, and then, that trace will be a line.

3. *The two Extreams of a Line are Points.*

4. *A right Line is that, whose points are equally plac'd between the two Extreams.*

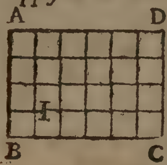
'Or thus. A right Line is the shortest that can be drawn from one point to another. Or yet. The Extreams of a right Line may cast a shadow upon the whole Line.

5. *A Superficies or Surface is a quantity, to which is attributed length, and breadth, without the consideration of any thickness.*

 6. The

6. *The Extreams of a Superficies are Lines.*

7. *A plane or right Superficies is that, whoſe lines are equally plac'd between its two Extreams; Or that, to which a right line may be every way apply'd.*



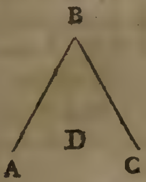
‘I have before obſerv'd, that motion may produce any quantity whatſoever: accordingly we ſay, when one line moves over another, it produces a Superficies, or a Plane; and that that motion has a kind of affinity with *Arithmetical* Multiplication. Suppoſe then the line AB to paſs along the line BC, retaining ſtill the ſame ſituation, without any inclination to one ſide or other: the point A will deſcribe the line AD, the point B the line BC, and the intermediate points the lines parallel to thoſe, which will make up the Superficies ABCD. I add further, that this motion anſwers to *Arithmetical* Multiplication; becauſe did I know the number of points, that are contain'd in both thoſe lines, AB, and DC; by multiplying them together, I ſhould find a product, which would give me the number of points, which conſtitute the whole ſuperficies ABCD. As for example, if AB contain'd four points, and BC ſix, by ſaying four times ſix make twenty four, I find, that the whole ſuperficies ABCD conſiſts of twenty four points. Now

‘by

'by a *Mathematical* point, may be underſtood
'any quantity whatſoever; *e. g.* a Foot, provi-
'ded it be not ſubdivided into parts.

8. *A plain Angle is the * diſtance or opening of two lines touching each other, ſo as not to compoſe only one line.*

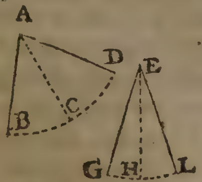
**Overture.* Gall. πρὸς ἀλλήλας κλίσις. Eucl.



'As the diſtance D betwixt the
'lines AB, and BC; which are not
'parts of the ſame line.

9. *A Rectilineal Angle is the diſtance betwixt two right lines.*

'Tis chiefly of this ſort of An-
'gles that I would be underſtood at preſent;
'which I define by *diſtance or opening*, becauſe
'Experience teaches, that the greateſt part of
'Beginners deceive themſelves in meaſuring the
'greatneſs of an Angle by that of the lines,
'within which it is contain'd.



'The Angle that is more
'open, is the greater; that
'is, when the lines of one an-
'gle lie more apart from each
'other than thoſe of another,
'taking them at the ſame di-
'ſtance from the points of concourſe, the for-
'mer is greater than the latter. Accordingly,
'the angle A is greater than the angle E; be-
'cauſe

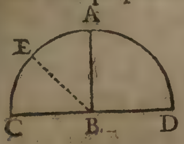
'cauſe taking the points D and B as remote 'from the point A, as the points G and L are 'from the point E; the points D and B lie far-'ther apart from each other, than the points G 'and L: from whence I infer, that if the lines 'EG and EL were produc'd farther, the angle 'E would be always of the ſame largeneſs, and 'always leſs than the angle A.

'We uſe three letters when we ſpeak of an 'Angle, of which the middlemoſt denotes the 'point of concourſe: as the angle BAD is the 'angle which by the lines BA and AD is form'd 'at the point A: the angle BAC is that made 'by the lines BA and AC: the angle CAD is 'compris'd by the lines CA and AD.

'A Circle is the meaſure of an Angle. There-'fore to know the magnitude of the Angle 'BAD, I place the foot of the Compaſs upon 'the point A, and deſcribe the circle BCD: 'the angle is ſo much the greater, by how 'many more parts of a circle the arch, that mea-'ſures it, contains: and becauſe a circle is uſu-'ally divided into 360 parts, or degrees, there-'fore an angle is ſaid to have twenty, thirty, 'forty degrees, according as the arch, com-'pris'd betwixt the lines that form it, contains 'ſo many. So the angle is the greater, which 'contains more degrees, as the angle BAD is 'greater than the angle GEL. 'The line CA 'divides the angle BAD in the middle, becauſe the

'the arches BC and BD are equal; and the an-
'gle BAC is part of the angle BAD, becauſe
'the arch BC is part of the arch BD.

10. *When one line falling upon another makes two equal angles, they are both right angles; and the line perpendicular.*



'As for example: if the line 'AB, plac'd upon the line CD, make the angles ABC and ABD equal; that is, if, having deſcrib'd a ſemicircle 'CAD from the center B, the arches AC and 'AD are equal: the angles ABC, and ABD 'are call'd right angles, and the line AB per-'pendicular. Therefore becauſe the arch CAD 'is a ſemicircle, the arches CA and AD are 'each of them a quarter of a circle, that is, the 'fourth part of three hundred and ſixty degrees, 'that is, ninety.

11. *An Obtuſe angle is that which is greater than a right one.*

'As the angle EBD is an obtuſe or blunt an-'gle, becauſe its arch EAD contains more than 'a quarter of a circle.

12. *An Acute angle is that which is leſs than a right one.*

'As the angle EBC is an acute, becauſe the 'arch EC, which meaſures it, has leſs than 'ninety degrees.

13. *A Term is the extremity or end of any quantity*

14. *A Figure is a quantity comprehended by one or more Terms.*

'That which is call'd a Figure ought to be 'limited and inclos'd on every side.

15. *A Circle is a plain figure, terminated by the encompassing of one line, which is call'd the Circumference; and is every where equally remote from the middle point.*

'The Figure RVSX is a 'Circle, because all the 'lines TR, TV, TS, TX, 'drawn from the point T, 'to the line RVSX, are e'qual.

16. *The middle point is call'd the Center.*

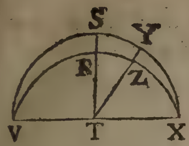
17. *The Diameter of a Circle is any line passing through the Center, and terminated at the Circumference, dividing the Circle into two equal parts.*

'As the lines VTX, and RTS.

'But if any should doubt, whether the line 'VTX does indeed divide the circle into two 'equal parts, so that the part VSX be equal to 'the part VRX; it may on this manner be 'prov'd.

'Suppose the part VRX to be plac'd upon the 'other VSX: I say, they will not exceed one

'the

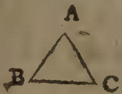


'the other. For if one, suppose 'VSX exceed the other VRX, 'the line TR will be less than 'TS; and in like manner TZ 'than TY, which is contrary to 'the definition of a Circle, which affirms all the 'lines drawn from the center to the circumfe-'rence to be equal.

18. *A Semicircle is a figure terminated by the Diameter, and half the Circumference.*

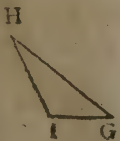
19. *Rectilineal figures are such as are terminated by right lines, having three, or four, or five, or as many sides as you please.*

'*Euclid* divides Triangles with respect either 'to their angles, or sides.



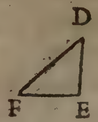
20. *An Equilateral Triangle is that which has its three sides equal: ABC.*

21: *An* Isosceles, *or equicrural Triangle, is that which has two sides equal:* 'As if the two sides AB, and AC be 'equal, 'the triangle ABC is an Iso-'sceles.



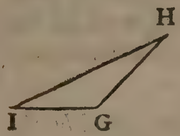
22. *A* Scalenum *is a triangle having all the three sides unequal, as GHI.*

23. *A*

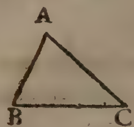


23. *A Rectangle triangle is that which has one right angle.*

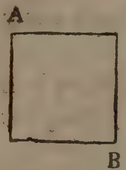
‘ As DEF, ſuppoſing the angle E to ‘ be a right one.



24. *An* Ambligone, *or Obtuſangle triangle is that which has one angle obtuſe.* As IGH.

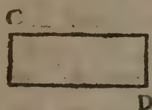


25. Oxygone, *or an Acutangle triangle is that whoſe angles are all acute.* As ABC.



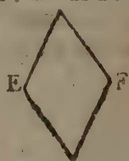
26. *A Rectangle* (properly ſo call’d) *is a figure conſiſting of four ſides, and having all its angles right.*

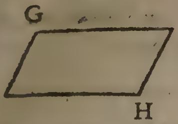
27. *A Square has all its ſides equal, and its angles right, as* AB.



28. *An Oblong Rect-angle has its ſides unequal, but its angles right: as* CD.

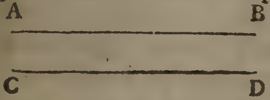
29. *A* Rhombus, *or Loſange, has equal ſides, but unequal angles: as* E F.



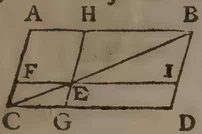


30. *A* Rhomboides, *or oblong Losange, hath both its sides and angles unequal: as* GH.

31. *Other irregular figures of four sides are call'd* Trapesia.



33. *Parallel lines are such, as being in the same plane will never concur, keeping still an equal distance one from the other: as AB, CD.*



33. *A Parallelogram is a figure, whose two opposite sides are Parallels:* 'as the Figure 'ABCD, whose sides AB, CD; 'and AC, BD, are parallels.

34. *The Diameter of a Parallelogram is a right line drawn from one angle to another: as* BC.

35. *The Complements are the two small Parallelograms, through which the Diameter does not pass: as AFEH, and GDIE.*

DEMANDS, *or* SUPPOSITIONS.

1. TIS suppos'd that a right line may be drawn from any point whatsoever to another.

2. That a right line may be continu'd to what length you please.

3. That

3. That from a Center given a Circle may be defcrib'd at any diftance whatfoever.

MAXIMS, *or* AXIOMS.

1. THofe quantities that are equal to a third, are equal betwixt themfelves.

2. If equal quantities be added to thofe that are equal, the products will alfo be equal.

3. If equal quantities be taken away from thofe that are equal, the remainders will be equal.

4. If you add equal parts to quantities unequal, they will remain unequal.

5. If from equal quantities you take away unequal parts, the remainders will be unequal.

6. Quantities that are double, triple, quadruple, *&c.* in refpect of the fame, are equal among themfelves.

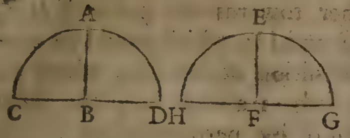
7. Thofe quantities are faid to be equal, which being apply'd one to the other, neither exceeds.

8. Equal lines and angles being plac'd one upon another, do not furpafs each other.

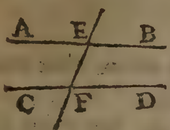
9. The whole is greater than its part.

10. All right angles are equal to one another.

'Let



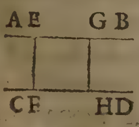
‘Let the two ‘right angles propos’d be ABC, ‘EFH, I say they ‘are equal. For if ‘two equal circles CAD, HEG, be describ’d ‘from the centers B and F; the fourth parts of ‘those circles CA, HE, which are the measures ‘of the angles, ABC, EFH, will be equal: ‘therefore the angles ABC, EFH, having ‘equal measures, will be equal.



The eleventh Maxim of *Euclid* is to this effect. If two lines AB, CD, being cut by a third EF, make the internal angles, BEF, DFE, less than two right angles; the lines AB, CD being produc’d, will at length concur towards the points B and D.

‘Which, though it be true, is not clear ‘enough to be receiv’d for a Maxim: therefore ‘I have substituted another in its place.

11: If two lines be parallel, all the perpendiculars contain’d betwixt them will be equal.



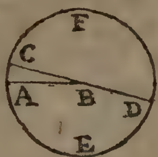
‘As for example, if the lines AB, ‘CD, are parallels, the perpendi‘cular lines FE, HG, are equal. ‘For if EF was greater than GH, ‘the lines AB and CD, would be more remote

 from

'from each other towards the points E & F, 'than towards G and H; which would be con'trary to the definition of Parallels, where tis 'ſaid, they are ſuch as always keep the ſame 'diſtance, meaſur'd by perpendiculars.

12. Two right lines cannot encloſe any ſpace; that is to ſay, they cannot encompaſs it on all ſides.

13: Two right lines cannot have one common ſegment.



'By which I mean, that two 'right lines, *ſuppoſe* A B, and 'CB, meeting at the point B, 'cannot together make one ſole 'line BD; but cutting one an'other ſeparate again immedi'ately after their rencounter. For, if you de'ſcribe a circle from the point B as a center, 'AFD will be a ſemicircle, becauſe the right 'line ABD, paſſing through the center B, will 'divide the circle into two equal parts. The 'ſegment CFD will be alſo a ſemicircle, be'cauſe CBD will be alſo a right line, and will 'paſs through the center B: therefore the ſeg'ment CFD will be equal to the ſegment AFD. 'the part to the whole; which is repugnant 'to the ninth Maxim.

Adver-

Advertisement.

'There are two ſorts of Propoſitions: In 'ſome we have nothing but the bare Spe-'culation of a Truth, without deſcending to 'Practice, which we call *Theorems*; in the 'other ſomething is propos'd to be done, and 'thoſe are call'd *Problems.*

'The firſt number of the quotations denotes 'the Propoſitions, the ſecond the Book. As '*by the* 2 *of the* 3. that is, by the ſecond Pro-'poſition of the third Book: but if only one 'number occur, it ſignifies ſuch a Propoſition 'of the book you are then upon.

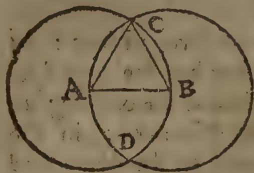
PROPOSITION I.

A Problem.

To draw an Equilateral Triangle upon any line given.

LET the line AB be propos'd for the baſe of an Equilateral Triangle; from the center A at the diſtance AB deſcribe the circle BCD; and likewiſe from the center B at the diſtance BA deſcribe the circle DAC cut-

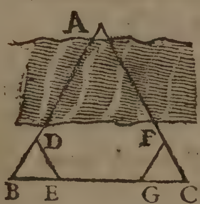
ting the former at the point C. Then draw the lines AC and BC; and all the ſides of the triangle ABC will be equal.



Demonſtration

The Lines AB, and AC being drawn from the ſame center A to the circumference of the circle BCD are equal, *by the Definition of a Circle*; the lines BA, and BC are likewiſe equal being drawn from the center B to the circumference of the circle CAD. Laſtly the lines AC and BC being equal to the ſame line AB, are alſo equal between themſelves. All the three ſides therefore of the triangle ABC are equal.

The USE.



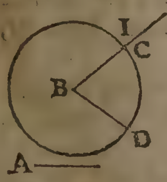
'The deſign of *Euclid* in 'placing this Problem here 'was only to demonſtrate the 'two following Propoſitions. 'But it may be alſo further 'ſerviceable for the meaſuring 'an inacceſſible line, as for example, the line 'AB, which by reaſon of a River or Precipice 'cannot be approach'd. In ſuch a caſe make 'a ſmall Equilateral Triangle BDE, either of 'Wood, or Copper, or the like; and having 'placed it Horizontally upon B, obſerve the point

'point A, by the ſide BD, and any other point 'C, by the ſide BE. Then transfer your Triangle along the line BC, and place it upon 'divers parts of the ſame line; till at length 'you find a point C, upon which placing the 'Triangle you ſhall ſee the point B, by the ſide 'CG, and the point A by the ſide CF. I ſay 'the lines CB and CA are equal; ſo that by 'meaſuring the line BC, you may know the 'line AB. I might further demonſtrate that 'the lines AB, and BC are equal; but let it 'ſuffice that in this Propoſition you are taught 'the way of making an Inſtrument proper to 'take the dimenſions of an inacceſſible line.

PROPOSITION II.

A PROBLEM.

From a point given to draw a line equal to another line given.



LET the point propos'd be B, from which a line is to be drawn equal to the line A. Take with the Compaſs the length of the line A, and at that interval, making B the Center, deſcribe the circle CD. Drawing then from

the point B to which ſide you pleaſe, a line BI, or BD, 'tis evident it will be equal to the line A.

' *Euclid* propoſes a more myſterious and in-
' tricate method of demonſtrating this Pro-
' poſition; but in practice we always make uſe
' of this; in as much as, having taken with the
' compaſs the line A, 'tis as eaſie deſcribing a
' circle from the center B, as from the cen-
' ter A.

PROPOSITION III.

A Problem.

From a greater line to take a part equal to a leſs.

SUppoſe you were to take from the line BC, a part BI, equal to the line A. Take betwixt the points of the compaſs the length of the line A, and at that diſtance, from the center B deſcribe a circle, which ſhall cut the line BC at the point I. 'Tis certain the lines BI, and A, are equal.

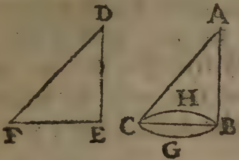
' The *Uſe* of theſe two preceding Propoſi-
' tions is ſufficiently evident; for as much as
' we are frequently oblig'd in *practical Geome-*
' *try* to draw one line equal to another; and to
' take a part of a greater line equal to a line that
' is leſs.

PROP-

PROPOSITION IV.

A THEOREM.

If two Triangles have two ſides equal, each to the other reſpectively, and the angles alſo, form'd by thoſe two ſides, equal; their baſes and other angles will be equal.



LET the triangles ABC, DEF, have two ſides equal each to the other reſpectively; that is to ſay, let AB be equal to DE, and AC to DF; and let the angles BAC, EDF, form'd by thoſe ſides be alſo equal; I ſay, the baſes BC, EF, are equal, and the angles ABC, DEF; ACB, DFE, are equal; and laſtly, the whole triangles equal in all reſpects,

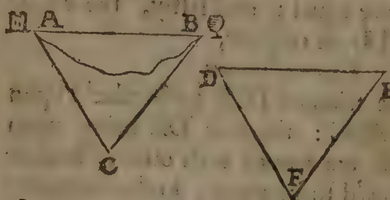
Demonſtration.

Suppoſe the triangle DEF to be plac'd upon the triangle ABC: the ſide DE being upon AB, they will not exceed each other, becauſe they are ſuppos'd to be equal; ſo that the point E will be upon B, and the point D upon the point A. For the ſame reaſon the line DF will fall upon AC. For if it ſhould fall on the out-

ſide of it, the angle EDF would be greater than the angle BAC; and if it ſhould fall within AC, the angle EDF would be leſs: and yet they are ſuppos'd to be equal. Therefore ſince the point D is upon the point A, and the line DF falls upon the line AC, to which it is equal, they will not exceed each other, but the point F will fall upon C. Laſtly, ſince the points E and F of the line EF, fall upon B and C; the line EF will fall upon BC; becauſe it can neither fall higher as in BHC, nor lower as in BGC; for then two right lines would encloſe ſpace; which is contrary to the twelfth Maxim. Therefore, the two triangles do not at all exceed each other; but not only the baſes BC, EF, but alſo the angles ABC, DEF; and ACB, DFE, are equal.

Coroll. An Equilateral triangle hath all its angles equal.

The USE.

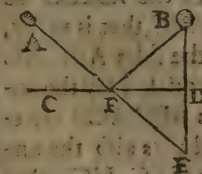


'Suppoſe I 'was to meaſure an inacceſſible 'line AB. I 'obſerve 'from the point C the points A, and B; and 'then meaſure the angle C. This done, placing 'a Board horizontally, and obſ:rving ſucceſſive-

'ly

'ly by a rule the points A and B, I draw two 'lines according to the rule, which make the 'angle C; and meaſure with a yard the lines 'AC, and BC, which are ſuppos'd acceſſible. 'Then going into an open field, and placing 'my Board again horizontally upon the point 'F, and obſerving the lines that I drew upon it, 'I make an angle DFE equal to the angle C; 'I make likewiſe FD, FE, equal to CA, CB, 'Then according to this Propoſition the lines 'AB, and DE, are equal. So that meaſuring 'by the yard the acceſſible line DE, I ſhall 'know AB, which is inacceſſible.

Another USE.



'The ſame propoſition may 'ſerve to teach how to hit a 'bowl at Billiards by refle-'ction. Suppoſe one bowl to 'be at the point A, and that 'which you would hit at the 'point B, and CD the Billiard table. Imagine 'then a perpendicular BDE, and take the 'line DE equal to BD. I ſay, if you direct 'the bowl from the point A to E, the reflexion 'will carry it to B. For in the triangles BFD, 'EFD, the ſide FD being common, and the 'ſides BD and DE equal; the angles BFD, 'EFD are equal, by this propoſition. The an-

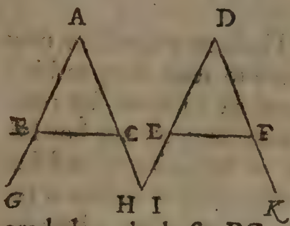
'gles

'gles AFC, DFE, being oppoſite, are alſo equal, 'as I ſhall demonſtrate hereafter. Therefore 'the angle of Incidence AFC, is equal to the 'angle of Reflection BFD; and by conſequence the Reflection will be by AFB.

PROPOSITION V.

THEOREM.

In Iſoſceles, or Equicrural triangles, the angles that are above the Baſe are equal: as alſo thoſe that are below it.



LEt the Iſoſceles be ABC, that is to ſay, let the ſides AB, AC be equal. I ſay the angles ABC, ACB are equal; as alſo the angles GBC, HCB, that are below the baſe BC. Suppoſe another triangle DEF, having the angle D equal to the angle A; and the ſides DE, DF, equal to AB, AC. Since the ſides AB, AC are equal, all the four lines AB, AC, DE, DF will be equal.

Demonſtration. Since the ſides AB, DE; AC, DF, are equal; as alſo the angles A, and D: if the triangle DEF be plac'd upon ABC, they will

will not exceed each other, but the line DE will fall upon AB; DF upon AC; and EF upon BC (by the *4th.*) therefore the angle DEF, will be equal to ABC. And becauſe one part of the line DE falls upon AB, the whole line DI will be upon AG; otherwiſe two right lines would have a common ſegment; therefore the angle IEF will be equal to GBC. Suppoſe then the triangle DEF turn'd, and apply'd another way to the triangle ABC, that is to ſay, ſo as DF may fall upon AB, and DE upon AC. Since the four lines AB, DF; AC, DE, are equal; as alſo the angles A and D: the triangles will likewiſe agree this way, and the angles ACB, DEF; HCB, IEF, will be equal. Now by the firſt comparing them it appear'd, that the angle ABC was equal to the angle DEF; GBC to IEF: therefor the angles ABC, ACB being equal to the ſame DEF; and GBC, HCB, alſo equal to the ſame IEF, they are equal among themſelves.

'I was unwilling to make uſe of *Euclid's* 'demonſtration, becauſe being very difficult, 'it might diſcourage beginners,

PROP-

PROPOSITION.

THEOREM.

If two angles of a triangle be equal, the triangle will be an Isosceles.

LET the angles ABC, ACB of the triangle ABC be equal: (*see Fig. preced.*) I say it is an Isosceles; that is to say, the two sides AB, AC, which are opposite to the equal angles, are equal. Suppose the triangle DEF to have a base EF equal to BC, and the angle DEF equal to ABC, as also DFE equal to ACB: since the angles ABC, ACB are suppos'd to be equal, all the four angles ABC, ACB, DEF, DFE, will be equal. Suppose again therefore the base EF to be plac'd upon the base BC, so that the point E lie upon the point B, the bases being suppos'd equal it is evident they will not exceed each other. Further, the angle E being equal to the angle B, and the angle F to the angle C; the line ED will fall upon the line BA, and FD upon CA: so that the lines ED and FD will meet at the point A. From whence it follows, that the line EC is equal to BA.

Let then the triangle DEF be turn'd to the other side, and be applied another way to the tri-

triangle ABC: that is to say, so that the point E lie upon C, and F upon B: the bases BC, FE will perfectly agree, being suppos'd to be equal: and because the angles F, and B; E, and C, are also suppos'd to be equal, the side FD will fall upon BA, and ED upon CA; and the point D upon A. Therefore the lines AC, DE will be equal. Whence it follows, that the sides AC, AB are equal between themselves, being equal to the same side DE.

The USE.

'This Proposition may serve 'for taking the dimensions of 'any sort of inaccessible lines. ''Tis said that *Thales* was the 'first that measur'd the heighth 'of Obelisks by their sha-'dows: it may be done by this Proposition. 'For if you were to measure the height of the 'Obelisk AB; do but expect till the Sun be 'elevated 45 degrees above the Horizon; that 'is to say, till the angle ACB be 45 degrees; 'and, *by the sixth Proposition*, the shadow BC 'will be equal to the Obelisk AB. For since 'the angle ABC is a right angle, and the angle 'ACB half a right one, or of 45 degrees; the 'angle CAB will be half a right one, as I shall 'prove hereafter. Therefore the angles BCA,

'BAC,

'BAC, are equal: and (*by the* 6.) the ſides AB, 'BC, are alſo equal. I can alſo meaſure the ſame 'height without making uſe of the ſhadow, by 'taking a ſtand ſo far from the point B, as that 'the angle ACB may be half a right angle, 'which may be known by a *Quadrant.*

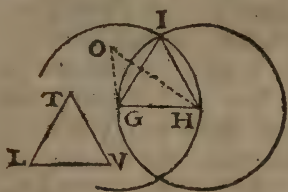
'Theſe Propoſitions are of frequent uſe in 'Trignometry, and in all other tracts.

'The *ſeventh* Propoſition may be omitted, 'becauſe 'tis of no other uſe but to demonſtrate 'the *eighth*, which may be done without it.

PROPOSITION VIII.

Theorem.

If two Triangles have all their ſides equal, their oppoſite angles will alſo be equal.



LET the ſide GI be equal to LT; HI, to VT; GH, to LV; I ſay, that the angle GIH, will be equal to the angle LTV; IGH, to the angle L; and IHG, to the angle V. From the center H, at the diſtance HI, deſcribe the circle IG; and from the center G, at the diſtance GI, the circle HI. *Demon-*

Demonstration.

Suppose the line LV brought upon HG: they would not exceed each other, because they are suppos'd to be equal. I add, that the point T will fall precisely upon the point I: For it ought to reach precisely to the circumference of the circle IG, because by the supposition the lines HI and VT are equal. It ought in like manner to reach to the circumference of the circle IH, because the lines GI and LT are equal. So then it will light upon the point I, being the point where those two circles cut each other. Indeed if it fell any where else, as upon O, the line HO, that is to say VT, would be greater than HI; and the line GO, that is LT, would be less than GI; which is against the supposition. Whence I conclude, that the triangles will exactly correspond, and the angle GIH be equal to the Angle LTV.

The USE.

'This Proposition is necessary for the proof 'of those that follow. And further, when we 'connot take the measure of an angle, because, 'the lines meeting in a solid body, we cannot 'apply our Instruments to it; we must take 'the three sides of the triangle, and make an'other upon a paper, whose angles we may 'measure. This is a very ordinary practice in

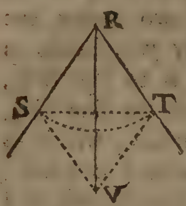
Gno-

'*Gnomonicks*, or Dialling; and in the Treatises
'concerning cutting precious stones, so as to fit
'the pannels, and to retain the waters.

PROPOSITION IX.

PROBLEM.

To divide an Angle into two equal parts.



LET the angle SRT be propos'd to be divided into two equal parts. Take the Compass, and from the center R, at any distance, draw the arch ST, cutting off two equal lines RS, RT. Then draw the right line ST, and (*by the* 1.) describe an equilateral triangle STV. I say, the line VR divides the angle into two equal parts: that is to say, the angles VRT, and VRS, are equal.

Demonstration.

The triangles VRS, and VRT, have the side VR common; and the side RT was taken equal to the side RS: the base also SV, is equal to VT, because the triangle SVT is equilateral. Wherefore (*by the* 8) the angles SRV, VRT, are equal.

The

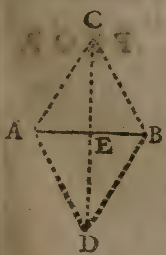
The USE.

'This Propoſition is very uſeful to divide 'the fourth part of a circle into degrees: for tis 'the ſame thing to divide an arch, as an angle 'into two equal parts; and the line RV does 'both, that is,it divides both the arch ST, and 'the angle SRT. Having therefore apply'd the 'ſemidiameter to the fourth part of a circle, 'you cut off an arch of 60 degrees, which divi'ded equally gives an arch of 30; and that 'again divided, makes one of 15 degrees. Tis 'true, to finiſh this diviſion, we muſt divide an 'arch into three equal parts, but that is not to 'be done Geometrically. Pilots alſo divide the 'Compaſs into 32 winds by the help of this 'Propoſition only.

PROPOSITION X.

A PROBLEM.

To divide a right line into two equal parts.



SUppoſe the line AB was to be divided into two equal parts; upon the line AB deſcribe an equilateral triangle ABC, (*by the* 1.) and divide the angle ACB into two equal parts by the line DC, (*by the* 9.) I ſay

the line AB is divided equally at the point E; that is to ſay, the lines AE and EB are equal.

Demonſtration.

The triangles ACE, and BCE have the ſide CE common, and the ſides CA and CB are equal, becauſe the triangle ACB is equilateral: and the angle ACB being divided equally, the angles ACE and BCE are alſo equal. Therefore (*by the* 4.) the baſes AE and BE are equal.

The USE.

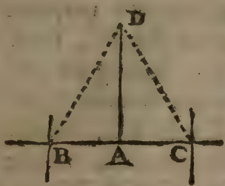
'Great uſe is made of this Propoſition, ordinary 'practices frequently requiring us to divide a 'line in the middle, which *Geometricians* re'quire ſhould be done exactly at the firſt daſh, 'by a method that is infallible, and not by eſ'ſays. This practice is likewiſe principally uſe'ful for dividing the parts.

PROP.

PROPOSITION. XI.

A PROBLEM.

To draw a Perpendicular to a line given, upon a point of the same line.



SUppose you were to raise a perpendicular upon the point A of the line BC. Take two equal lines AB and AC on both sides the point A, and make an equilateral triangle BDC upon the line BC, (*by the* 1.) I say the line AD is perpendicular, that is to say, the Angles BAD and CAD are equal.

Demonstration.

The triangles BAD, and CAD have the side AD common, the sides AC and AB are equal, and the bases BD and DC also equal: therefore (*by the* 8.) the angles BAD, and CAD, are equal; and (*by the* 10. *def.*) the line AD perpendicular to BC.

PROPOSITION. XII.

A PROBLEM.

To draw a perpendicular to a line given, from a point which is out of the line.

IF you would draw a perpendicular to the line BC, from the point A: having ſet the foot of the compaſs upon A, deſcribe the Circle BC, which ſhall cut the line BC, at the points B and C. Then divide the line BC into two equal parts at the point E. I ſay the line AE is perpendicular to BC. Draw the lines AB, AC.

Demonſtration. The triangles BEA, and CEA have the ſide AE common; and the ſides EC and EB equal, the line BC having been equally divided at the point E; the baſes AB and AC, being drawn from the center A to the circumference BC, are likewiſe equal: therefore the angles AEB, and AEC, are equal, (*by the* 8) and the line AE perpendicular, (*by defin.* 10.)

The method, in practice, of dividing the line BC in the middle, ir to deſcribe two arches at D, at the ſame interval, from the centers B and C.

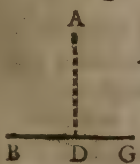
The USE.

'We have need of a Plummet or Squaring-'line almost in all our operations: no angles 'are of use in buildings but the right; and all 'chairs, benches, tables, buffets, and other move-'ables, are fram'd by the square. No survey of 'Land can be taken without making use of per-'pendicular lines; nor can *Dialling* be per-'form'd without them; The Carpenter's Le-'vel contains a right angle, and the same is 'preferr'd before any other, especially by the '*French*, in Fortifications. Lastly, not only '*Mathematicians*, but also the greatest part 'of practical Artisans, require that we should 'know how to draw a perpendicular.

PROPOSITION XIII.

A Theorem.

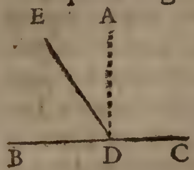
One line falling upon another makes with it either two right angles, or two angles equal to two right ones.



LEt the line AD fall upon BG; I say, 'twill make with it either two right angles; or two angles, one obtuse, and the other acute, which joyn'd together shall be of equal value with two right ones.

Demonſtration.

Suppoſe the line AD to fall perpendicularly upon BG, then tis evident (*by defin.* 10.) that the angles ADB, and ADG, are equal, and by conſequence right angles. Or.



Secondly, ſuppoſe the line ED not to fall perpendicularly upon BC, and draw a perpendicular AD (*by the* 11.) the angles ADB, and ADC are right angles, which are of equal value with the three angles ADC, ADE, EDB. But the obtuſe angle EDC, and the acute angle EDB, are of equal value with the three angles ADC, ADE, and EDB: therefore the angles EDC, and EDB, are of equal value with two right ones.

This Propoſition may be more eaſily demonſtrated by deſcribing a ſemicircle from the center D upon the line BC. For the angles EDB, and EDC, will require a ſemicircle for their meaſure, which is the meaſure of two right angles, as I have ſhewn before; *in the* 8 *definition.*

Coroll. 1. If the line AD falling upon BC, make one right angle ADC; it is evident the other, ADB, will be alſo a right angle.

Coroll. 2. If the line ED, falling upon BC, make the angle EDB acute; the angle EDC will be obtuſe.

The

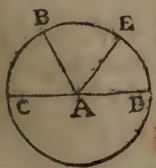
The USE.

'By this means, when we know one of the 'angles which is made by one line falling upon 'another, we know alſo the other: as for ex 'ample, if the angle EDB be one of 70 de- 'grees, taking away 70 from 180, there 'will remain 110 for the angle EDC. This 'operation does frequently occur in *Trigono- 'metry*; and alſo in *Aſtronomy*, for finding the 'eccentricity of the circle through which the 'Sun anually paſſes.

PROPOSITION XIV.

A THEOREM.

If two lines meeting together at the ſame point of another line, make with it two angles equal to two right ones; they will make but one and the ſame line.



SUppoſe the lines CA, and DA, to meet at the point A of the line AB; and that the angles adjoyning, CAB, and BAD, are equal to two right ones. I ſay, the lines CA and DA are but one and the ſame line; ſo that CA being continued, will fall preciſely upon AD.

Imagine, if you pleaſe, that CA continu'd will paſs on to E, and from the center A deſcribe a circle.

Demonſtration.

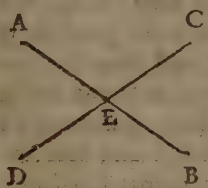
If you ſay that CAE is a right line, the arch CBE will be a ſemicircle. But 'tis ſuppos'd, that the angles CAB, and BAD are equal to two right ones, and that therefore their meaſure is a ſemicircle. Therefore the arches CBE, and CBD will be equal; which is impoſſible, one being a part of the other. Therefore the line CA being continu'd, will make but one and the ſame line with AD.

PROPOSITION XV.

A THEOREM.

*If two right lines cut each other, the oppoſite angles * at the top will be equal.*

* κατὰ κορυφὴν *Eucl.* au ſommet. *Gall.*



LET the lines AB and CD cut each other at the point E: I ſay, the angles AEC, and DEB, which are oppoſite at the top are equal.

Demonſtration.

The line CE falling upon the line AB, makes the

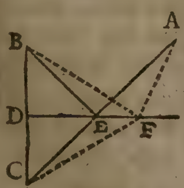
the angles AEC and CEB equal to two right ones, (*by the* 13.) In like manner the line BE falling upon the line CD, makes the angle CEB and BED equal to two right ones. Therefore the angles AEC, CEB, taken together, are equal to the angles CEB, BED; therefore taking away the angle CEB from both, the angle AEC will remain equal to DEB, (*by the* 3. *Maxim.*)

Coroll. 1. If two lines DE, and EC, concurring at the same point E of the line AB, form with it the opposite angles AEC, DEB equal; DE and EC make but one right line.

Demonstration.

The line EC falling upon the line AB, makes the angles AEC, and CEB equal to two right ones, (*by the* 13.) 'Tis supposed likewise that the angle DEB is equal to the angle AEC. Therefore the angles DEB, BEC, are equal to two right ones. And (*by the* 14.) the lines CE and ED make but one right line.

Another USE.



'The two preceding Propositions are made use of to 'prove, that two lines make 'but one total. As for example, in Catopcricks or Perspe-'ctives, where that is required 'to prove, that all the lines that

'that can be drawn by reflexion from the point
'A to the point B, thoſe are the ſhorteſt,
'which make the angle of Incidence equal to
'the angle of Reflexion. As for example; if the
'angles BED and AEF be equal, the lines AE,
'and EB, are ſhorter than AF, and FB. From
'the point B draw a perpendicular BD, and
'make the lines BD and CD equal; then draw
'EC, and FC. Firſt in the triangles BED and
'CED the ſide DE is common; and the ſides
'BD, and DC being equal, as alſo the angles
'BDE, and CDE; the baſes BE, and CE will
'be equal; as alſo the angles BED, and DEC,
'(*by the* 4.) In like manner I may prove, that
'BF, and CF are equal.

Demonſtration.

'The angles BED and DEC are equal, and the
'angles BED and AEF are ſuppos'd likewiſe to
'be equal; therefore the oppoſite angles DEC
'and AEF will be equal; and (*by the Coroll. of
the* 15.) AEC one right line; and by con-
'ſequence AFC is a triangle, of which the ſides
'AF and FC muſt be longer than AEC, that is
'to ſay, than AE, and EB. But the lines AF
'and FC are equal to the lines AF and FB;
'therefore the lines AF and FB are longer than
'the lines AE and EB. And ſince natural cau-
'ſes always act by the ſhorteſt lines, the Refle-
'xion will always happen in ſuch a manner, that
'the angles of Reflexion and Incidence may be
'equal.

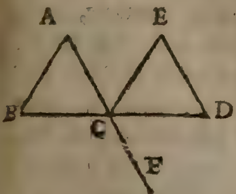
Fur-

‘ Further, because we can easily prove, that ‘ all the angles that can be made upon a plane ‘ about the same point, are equal to four right ‘ angles; for as much as in the first figure of ‘ this proposition, the angles AEC and AED ‘ are equal to two right ones, as also BEC and ‘ BED to two more: we make a general rule ‘ to determine what Polygones may be joyn'd in ‘ paving a Hall. Accordingly we say, that four ‘ squares, six triangles, and three hexagones, ‘ may be used for that purpose; and that there-‘ fore Bees are always observ'd to make their ‘ little cells of the last, that is, of figures consist-‘ ing of six sides.

PROPOSITION XVI.

A Theorem.

The external angle of a triangle is greater than either of the internal opposite angles.



PRoduce the side BC of the triangle ABC I say the external angle ACD, is greater than either of the internal opposite angles, ABC, or BAC. Suppose the triangle ABC to be mov'd along the line BD,

and

and carry'd into the place of CED.

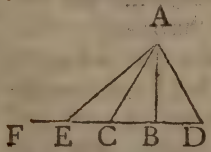
Dmonstration.

'Tis impossible that the triangle ABC should be so mov'd, but the point A must come into the place of the point E; and then t'will appear, that the angle ECD, that is to say, ABC, is less than the angle ACD: therefore the internal angle ABC is less than the external ACD.

'Tis likewise easie to prove, that the angle A is less than the external angle ACD: for having prolong'd the side AC as far as F, the opposite angles BCF, and ACD, are equal (*by the* 15.) Therefore causing the triangle ABC to slide along the line ACF, I shall demonstrate, the angle BCF to be greater than the angle A.

The USE.

We may draw from this proposition many 'most useful conclusions. As first, that from a 'point given only one perpendicular can be 'drawn to the same line. For example, Suppose the line AB to be 'perpendicular to the line 'BC: I say, that AC will 'not be perpendicular; because the right angle



'ABD must be greater than the internal angle 'ACB; therefore ACB cannot be a right angle, 'nor AC a perpendicular.

Se-

'Secondly, that from the same point A can-
'not be drawn more than two equal lines; for
'example, AC, and AD; and if you draw a
'third as AE, it will not be equal to the for-
'mer. For since AC and AD are equal, the an-
'gles, ACD and ADC, are equal, (*by the 5.*)
'but in the triangle AEC, the external angle
'ACB is greater than the internal AEC: and
'therefore likewise the angle ADE, is greater
'than the angle AED. Therefore the lines AE,
'and AD, are not equal; nor by consequence
'AC and AE.

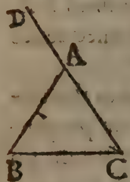
'Thirdly, that if the line AC makes the an-
'gle ACB acute, and ACF obtuse, the perpen-
'dicular drawn from the point A will fall on
'the side of the acute angle. For if you say
'that AE is a perpendicular, and that AEF is a
'right angle; the right angle AEF would be
'greater than the obtuse ACE. These conclu-
'sions are serviceable for measuring Parallelo-
'grams, Triangles, and Trapesia, and to re-
'duce them into rectangular figures.

PROP.

PROPOSITION XVII.

A Theorem.

Any two Angles of a triangle are leſs than two right ones.



LET the triangle be ABC; I ſay, that any two of its angles taken together, as BAC, and BCA, are leſs than two right ones. Produce the ſide CA to the point D.

Demonſtration.

The internal angle C, is leſs than the external BAD, (*by the* 16.) Add therefore to both the angle BAC; the angles BAC, and BCA, will be leſs than the angles BAC, and BAD; yet thoſe are but equal to two right ones, (*by the* 13.) therefore the angles BAC, and BCA, are leſs than two right ones.

After the ſame manner I can demonſtrate the angles ABC, and ACB, to be leſs than two right ones, by producing the ſide BC.

Coroll. If one angle of a triangle be a right, or obtuſe angle, the others are acute.

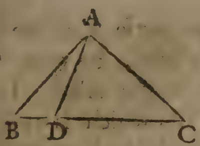
" This Propoſition is neceſſary to demonſtrate thoſe that follow.

PROP.

PROPOSITION XVIII.

A THEOREM.

In every triangle whatsoever the greatest side is oppos'd to the greatest angle,



Suppose the side BC of the triangle ABC, to be greater than the side AC: I say, the angle BAC, that is oppos'd to the side BC, is greater than the angle B, which is oppos'd to the side AC. Cut the line BC in D, so that CD may be equal to AC; then draw the line AD.

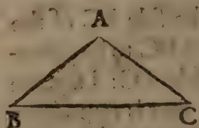
Demonstration.

Since the sides AC, and CD, are equal, the triangle ACD will be an *Isosceles*, and (*by the* 5.) the angles CDA, and CAD, equal. Now the whole angle BAC is greater than the angle CAD: therefore the angle BAC is greater than the angle CDA; which yet, being an external angle in respect of the triangle ABD, is greater than the internal B, (*by the* 16.) Therefore the angle BAC is greater than the angle B.

PROPOSITION XIX.

A THEOREM.

In every triangle the greatest angle is oppos'd to the greatest side.



LET the angle A of the triangle BAC, be greater than the angle ABC. I say, the side BC which is oppos'd to the angle A, is greater than the side AC, that is oppos'd to the angle B.

Demonstration.

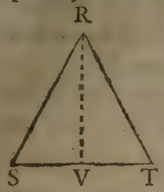
If the side BC be not greater than the side AC, 'tis either equal; and then the angles A and B would be equal, (*by the* 5.) which is contrary to the supposition: or less; and if so, the side AC, being greater than BC, the angle B would be greater than the angle A, though the contrary be suppos'd. It remains therefore that the side BC be greater than the side AC.

The USE.

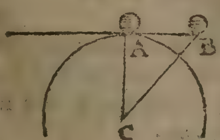
' We may prove from these propositions ' not only that no more than one perpendicular

can

‘can be drawn from the ſame point to the ſame line; but alſo that it is the ſhorteſt of all. As for example; if the line RV be perpendicular to ST, it will be leſs than RS: becauſe the angle RVS being a right angle, the angle RSV will be an acute, (*by the Coroll. of the* 17.) and the line RV will be leſs than RS, (*by the preceding.*) Therefore *Geometricians* do always make uſe of a perpendicular, when they take the dimenſions of any thing, and reduce irregular figures to ſuch as have one or more right angles. I add, that it being impoſſible that more than three perpendiculars ſhould meet at the ſame point, it cannot be imagin'd that there ſhould be more than three ſpecies or kinds of quantity, a line, a ſuperficies, and a ſolid body.



‘By theſe propoſitions we likewiſe prove, that a bowl exactly round cannot reſt, but upon ſuch a certain point. For example; let the line AB repreſent a plane, and C the center of the earth, and that CA be drawn perpendicular to the line AB; I ſay, that a bowl being plac'd upon the point B, cannot reſt there. For a heavy body cannot reſt, when it may deſcend. Now



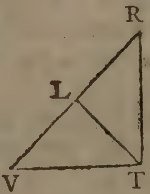
‘ the bowl B moving towards A continually de-
‘ scends, and approaches nearer the center of
‘ the earth C; because in the triangle CAB, the
‘ perpendicular CA is shorter than BC.

‘ In like manner we prove, that a liquid body
‘ must flow from B to A, and that its superfi-
‘ cies must be round.

PROPOSITION. XX.

A THEOREM.

Any two sides of a triangle taken together are greater than the third.



I say that the two sides TL, LV, are greater than the side TV. Some men prove this Proposition by the definition of a right line, which is the shortest that can be drawn from one point to another: therefore the line TV, is less than the two lines TL and LV.

But it may also be demonstrated another way. Continue the side VL to R, so that the lines LR, and LT be equal; then draw the line RT.

Demonstration.

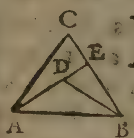
The sides LT, and LR, of the triangle LTR, are equal; therefore the angles R, and LTR, are equal, (*by the* 5.) But the angle RTV is greater

greater than the angle RTL: therefore the angle RTV is greater than the angle R: and *(by the 19.)* in the triangle RTV, the ſide RV, that is to ſay, the ſides LT and LV, are greater than the ſide TV.

PROPOSITION XXI.

A Theorem.

If a ſmall triangle be deſcrib'd within a greater, upon the ſame baſe, the ſides of the ſmall one will be leſs than thoſe of the greater; but they will form a greater angle.



LET the ſmall triangle ADB be deſcrib'd within the triangle ACB, upon the ſame baſe AB. I ſay firſt, the ſides AC and BC are greater than the ſides AD and BD. Continue the line AD to E.

Demonſtration.

In the triangle ACE, the ſides AC and CE, are greater than the ſide AE alone, *(by the 20.)* Therefore adding to them the ſide EB; the ſides AC, and CEB, are greater than the ſides AE, and EB. In like manner in the triangle DBE, the two ſides BE and ED are greater than the ſide BD alone, and adding the ſide AD,

 the

the ſides ADE, and EB, will be greater than AD and BD.

I ſay further, that the angle ADB is greater than the angle ACB: for the angle ADB is an external angle in reſpect of the triangle DBE, and therefore greater than the internal DEB (*by the* 16.) In like manner the angle DEB, being an external angle in reſpect of the triangle ACE, is greater than the angle ACE, therefore the angle ADB is greater than the angle ACB.

The USE.

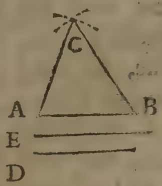
'By the help of this Propoſition we demonſtrate in *Opticks*, that the Baſe AB view'd from the point C, will appear leſs, than when it is beheld from the point D; according to that principle, That quantities view'd under a greater angle will appear greater. Therefore 'tis, that *Vitruvius* adviſes, not much to leſſen the tops of very high Pillars, becauſe they being ſo remote from our ſight, quickly appear ſlender enough without being diminiſh'd.

PROP.

PROPOSITION. XXII.

A Theorem.

To deſcribe a triangle, whoſe ſides ſhall be equal to three ſides given, provided that any two of them be greater than the third.



LET it be propos'd to deſcribe a triangle, whoſe ſides ſhall be equal to three lines given, AB, D, and E. Meaſure with the compaſs the line D, and ſetting one foot thereof upon the point B make an arch. Then take the line E, and placing the foot of your compaſs upon the point A, make another arch, cutting the former at the point C. Which done draw the lines AC, and BC. I ſay that the triangle ABC, is ſuch a one as you deſire.

Demonſtration.

The ſide AC is equal to the line E, becauſe it reaches to the arch, which is drawn from the center A at the diſtance of the line E; and for the ſame reaſon the ſide BC is equal to the line D: therefore the three ſides AC, BC, and AB, are equal to the lines E D, and AB.

I added a Proviſo, that the two lines ſhould be greater than the third: becauſe otherwiſe ſi

the lines D and E were leſs than the line AB, the Arches could not cut each other.

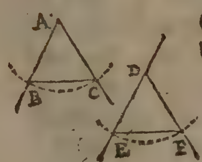
The USE.

'This Propoſition may be uſeful for deſcrib-
'ing a figure equal or like to another: for ha-
'ving divided that, which is propos'd to be
'equall'd or imitated, into triangles; and made
'other triangles, having equal ſides with the
'former; we ſhall have a figure exactly equal.
'But if we deſire only one that is like, but leſs;
'as when we would deſcribe a Plain, or Coun-
'try upon paper; having divided it into trian-
'gles, and meaſur'd all their ſides, we muſt
'make ſimilar triangles; giving to each of their
'ſides ſo many parts of a *Scale*, or line divided
'into equal parts, as the ſides of the triangles
'propos'd have of yards or feet.

PROPOSITION XXIII.

A PROBLEM.

To make an angle equal to another at a point of a line given.



SUppoſe you were to make an angle at the point A of the line AB, equal to the angle EDF. Deſcribe from the points A and D as centers two

wo arches BC, and EF, at the ſame wideneſs of the compaſs; then take the diſtance EF, and having meaſur'd as much at BC, draw the line AC. I ſay the angles BAC, and EDF, are equal.

Demonſtration.

The triangles ABC, and DEF, have the ſides AB, and AC, equal to the ſides DE, and DF; ſince the arches BC and EF were deſcrib'd with the ſame wideneſs of the compaſs: the baſes alſo BC and EF are equal, therefore the angles BAC and EDF are equal, (*by the* 8.)

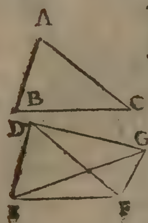
The U S E.

This Problem is ſo neceſſary in *Geodæſia*, *Fortifications*, *Perſpective*, *Dialling*, and all other parts of the Mathematicks, that the greateſt part of their Operations would be impoſſible, if we did not know how to make one angle equal to another, or of ſuch a number of degrees as we pleaſe.

PROPOSITION XXIV.

A THEOREM.

Of two triangles, having each two sides equal to two of the other, that which has the greatest angle, has also the greatest base.



LET the triangles ABC, DEF, have the sides AB and DE; AC and DF equal; and let the angle BAC be greater than the angle EDF. I say, the base BC is greater than the base EF.

Make the angle EDG equal to the angle BAC, (*by the* 23) and the line DG equal to AC; then draw the line EG. First the triangles ABC and EDG, having the sides AB and DE, AC and DG, equal, and the angle EDG, equal to the angle BAC; their bases BC and EG will be equal (*by the* 4.) and the lines DG and DF being both equal to AC, will be equal betwixt themselves.

Demonstration.

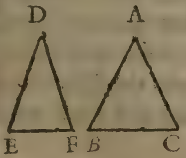
In the triangle DGF, the sides DG and DF being equal, the angles DGF and DFG will be equal, (*by the* 5.) But the angle EGF is less than the angle DGF, and the angle EFG is greater than the angle DFG. Therefore in the

the triangle EFG, the angle EFG will be greater than the angle EGF: and therefore (*by the* 18.) the line EG oppos'd to the greater angle EFG, will be greater than EF. Therefore BC, being equal to EG, is greater than EF.

PROPOSITION XXV.

A Theorem.

Of two triangles, having each two ſides equal to two of the other, that which has the greateſt baſe, has likewiſe the greateſt angle.



LET the two triangles ABC, DEF, have the ſides AB, DE; and AC, DF, equal; and let the baſe BC be greater than the baſe EF. I ſay, that the angle A will be greater than the angle D.

Demonſtration.

If the angle A be not greater than the angle D; it will be either equal, and then the baſes BC, EF, will be equal, (*by the* 4.) or it will be leſs, and the baſe EF greater than the baſe BC, (*by the* 24.) but both are contrary to the ſuppoſition,

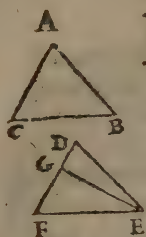
" Theſe Propoſitions are neceſſary to de-
" monſtrate thoſe that come after.

PROP.

PROPOSITION XXVI.

A THEOREM.

If one triangle has one side, and two angles, equal to those of another triangle; 'tis equal to it in all respects.



LET the angles ABC, DEF; ACB, DFE, of the triangles ABC, DEF, be equal; and the sides BC, and EF, which are between those angles, also equal. I say, that the other sides are equal; for example, AC, and DF. Imagine, if you please, the side DF to be greater than AC, and cutting GF equal to AC draw the line GE.

Demonstration.

The triangles ABC, GEF, have the sides EF, BC; AC, GE, equal; the angle C is also suppos'd to be equal to F. Therefore (*by the* 4)the triangles ABC, GEF, are equal in all respects; and the angles GEF, and ABC, are equal. But we suppos'd the angles ABC, DEF, to be equal: and so, the angles DEF, GEF, would be equal: that is, the whole to the part; which is impossible. Therefore the side DE will not be greater than the side AC, nor AC greater than DF,

DF, becaufe the fame demonftration may be made in the triangle ABC.

Again, fuppofe the angles A and D, C and F to be equal; and alfo the fides BC, and EF, oppos'd to the angles A and D, to be equal. I fay, the other fides are equal. For if DF be greater than AC, cut GF equal to AC, and draw the line GE.

Demonftration.

The triangles ABC, GEF, having the fides EF, BC; FG, CA, equal, will (*by the* 4) be equal in all refpects; and the angles EGF, BAC, will be equal. But we fuppos'd, that the angles A and D were equal, therefore the angles D, and EGF, muft be equal, which is impoffible, fince the angle EGF, being the external angle in refpect of the triangle EGD, muft be greater than the internal D, (*by the* 16.) therefore the fide DF is not greater than AC.

The USE.

'*Thales* made ufe of this Propofition to mea-
'fure inacceffible diftances. For example: the

'diftance AD being
'propos'd, he would
'draw from the point
'A, the line AC per-
'pendicular to AD;
'then defcribing a femicircle at the point C,
'would meafure the angle ACD, and take an-
'other

'other equal to it on the other ſide, prolonging 'the line CB till it met with the line DA at the 'point B; and then demonſtrated the lines 'AD and AB to be equal; ſo that meaſuring 'the line AB, which was acceſſible, he could 'know the other which was not. For the two 'triangles ADC, and ABC, have the right an-'gles CAD, and CAB equal, the angles ACD, 'and ACB are alſo taken equal; and the ſide 'AC is common to both: therefore (*by the* 26.) 'the ſides AD and AB are equal.

A *LEMMA.*

A line which is perpendicular to one of two parallels, is alſo perpendicular to the other.

'Let the parallel lines be AB, and CD, and let EF be perpendicu-'lar to CD. I ſay, tis alſo perpendi-'cular to AB. Cut the line CF e-'qual to FD, and upon the points 'C and D raiſe two perpendiculars to CD, 'which, *by the definition of Parallels*, will be e-'qual to FE; then draw the lines EC and ED.

Demonſtration.

'The triangles CEF, and FED, have the 'ſide FE common, the ſides CF, FD equal, 'and the angles CFE and EFD, right, and by 'conſequence equal; therefore (*by the* 4.) the

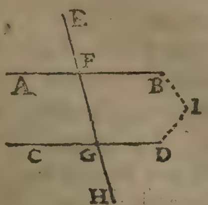
baſes

'bases EC, ED, and the angles FED, FEC, and 'FCE, FDE, will be equal; the two last of 'which being taken away from the right angles 'ACF and BDF, leave the two angles ACE, 'and BDE, equal; therefore the triangles 'CAE, DBE, will have (*by the* 4) the angles 'DEB, CEA, equal; which being added to the 'equal angles CEF, FED, make the angles 'FEB, and FEA, equal; therefore the line EF 'is perpendicular to AB.

PROPOSITION XXVII.

A Theorem.

If a line, falling upon two others, makes with them the alternate angles equal, those two lines are parallel.



LET the line EH, falling upon the lines AB and CD, make with them the alternate angles AFG, FGD equal, I say first, the lines AB, and CD, will never concur, though continu d as far as you please. For suppose them to concur in I, and that FBI, and GDI, be two right lines.

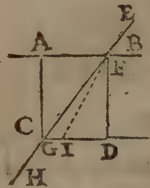
Demonstration.

If FBI, and GDI, be two right lines, FIG is

is a triangle; and (*by the* 16) the external angle AFG, is greater than the internal FGI. They cannot therefore be equal, if the lines AB and CD ever concur.

'But because we have Examples of some 'crooked lines, which never concur, and yet 'are not parallels, approaching still nearer and 'nearer to each other.

I say secondly, that if the line EH, falling upon the lines AB and CD, makes the alternate angles AFG, and FGD equal; the lines AB, CD, are parallel, or in all respects equally remote from each other, so that the perpendiculars between them will be equal. From the point G draw the perpendicular GA to the line AB; and taking GD equal to AF, draw FD.



Demonstration.

The triangles AGF, and FGD, have the side FG common; the side GD is also taken equal to the side AF, and the angles AFG and FGD suppos'd to be equal. Therefore (*by the* 4) the bases AG and FD are equal, and the angle CDF is equal to the right angle CAB; therefore FD is perpendicular. I add, that the line AB is parallel to CD: for the only parallel line that can be drawn from the point *F* to the line CD, ought to pass by the point A, according

cording to the definition of Parallels; which requires, that the perpendicular lines AG and FD be equal.

PROPOSITION XXVIII.

A Theorem.

If a line, falling upon two others, makes the external angle, equal to the internal opposite angle on the same side; or the two internal angles on the same side equal to two right ones; those two lines will be parallel.

IN the precedent figure, suppose the line EH, falling upon AB, and CD, to make first the external angle EFB equal to the internal opposite angle on the same side FGD. I say, that the lines AB, and CD, are parallel.

Demonstration.

The angle EFB is equal to the angle AFG, being oppos'd to it at the top *(by the 15.)* and 'tis suppos'd that the angle FGD is also equal to the angle EFB; therefore the alternate angles AFG, FGD, will be equal; and *(by the 27.)* the lines AB, and CD, will be parallel.

I say in the second place, that if the angles EFG, and FGD, which are the internal angles on the same side, be equal to two right ones, the lines

lines AB and CD will be parallel.

Demonstr. The angles AFG and BFG are equal to two right angles, (*by the* 13.) and 'tis suppos'd that the angles BFG, and FGD, are also equal to two right angles; therefore the angles AFG, BFG, are equal to the angles BFG, and FGD; therefore taking away the angle BFG, which is common to both, the alternate angles AFG and FGD will be equal; and (*by the* 27.) the lines AB and CD will be parallel.

PROPOSITION XXIX.

A Theorem.

If a line cut two parallels, the alternate angles will be equal; the external angle will be equal to the internal opposite angle; and the two internals on the same side will be equal to two right angles.

LET the line EH [*see fig. preced.*] cut the two parallels AB, and CD; I say first, the alternate angles AFG, and FGD, are equal. From the points F and G draw the perpendiculars GA, and FD, which by the definition of Parallels are equal.

Demonstration.

In the rectangle triangles AFG, and FGD, the

the ſides FD and AG being equal, as alſo the right angles A and D, and the ſide FG common to both. I ſay firſt, that the ſide GD is equal to AF. For if GD be greater; having cut the line DI equal to AF, and drawn the line FI; the triangles AFG, and FDI, would have their baſes GF and FI equal, which is impoſſible. For ſince the angle D is a right angle, the angle *FID* is an acute, and *FIG* an obtuſe, (*by the* 13.) therefore (*by the* 18.) in the triangle *FIG*, the ſide *FG* oppos'd to the obtuſe angle, is greater than *FI*. Therefore DG is equal to *AF*; and the triangles *AFG* and *FGD*, having all their ſides equal, will have the alternate angles *AFG* and *FGD* equal, as being oppos'd to the equal ſides AG, and *FD*.

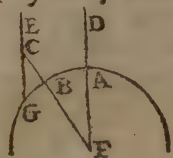
I ſay again, that the external angle *EFB* is equal to the internal EGD, becauſe (*by the* 15) it is equal to its oppoſite *AFG*, which is equal to its alternate *FGD*.

Laſtly, ſince the angles *AFG* and G*FB* are equal to two right ones; taking away *AFG*, and ſubſtituting in its place its alternate *FGD*, the two internal angles *GFB*, and *FGD*, will be equal to two right angles.

The USE.

'*Eratoſtenes* found out by theſe Propoſitions 'a way of meaſuring the circuit or circumfe-

‘ rence of the Earth. In order to which he ſup-
‘ pos’d two rays, proceeding from the center of
‘ the Sun to two points of the earth, to be phy-
‘ ſically parallel; and alſo that at *Syene*, a town
‘ in the higher parts of *Egypt*, the Sun comes
‘ exactly to the Zenith upon the day of the Sol-
‘ ſtice, obſerving the Wells there to be then
‘ illuminated to the very bottom: and likewiſe
‘ computed the diſtance between *Alexandria* and
‘ *Syene* by miles or furlongs.



‘ Let us therefore ſuppoſe *Sy-*
‘ *ene* to be at the point A, and
‘ *Alexandria* at B, where we
‘ erect a ſtyle BC perpendicu-
‘ lar to the Horizon; and let
‘ the two lines DF and EG re-
‘ preſent the two rays proceeding from the cen-
‘ ter of the Sun upon the day of the Solſtice,
‘ which are parallel to each other. DA,
‘ which paſſes by *Syene*, is perpendicular, that
‘ is, it paſſes through the center of the earth.
‘ Having obſerv’d by the perpendicular ſtyle
‘ BC the angle GCB, made by the ray of the
‘ Sun EG; I ſay, the rays DA and EG being pa-
‘ rallel, the alternate angles GCB and BFA are
‘ equal; by which means we have got the angle
‘ AFB, and its meaſure AB; which gives us in
‘ degrees the diſtance between *Alexandria* and
‘ *Syene*. And having ſuppos’d it to be known
‘ in miles, the circumference of the earth may

‘ be

'be found by the simple Rule of Three, saying,
'*If so many degrees give so many miles, how ma-*
'*ny will 360 give?*

PROPOSITION XIX.

A Theorem.

Lines parallel to a third, are also parallel among themselves.

Suppose the lines AB, and FE to be parallel to the line CD: I say, they are parallel betwixt themselves. Let the line GL cut them all three.

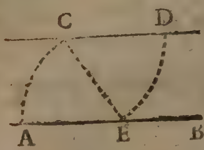
Demonstration.

For as much as the lines AB and CD are parallel, the alternate angles AHI, and HID, are equal (*by the* 29.) and because the lines CD and FE are also parallel, the external angle HID will be equal to the internal ILE; [*by the same*] therefore the alternate angles AHI, and ILE, will be equal, and the lines AB and FE parallel [*by the* 27.]

PROPOSITION XXXI.

A Problem.

To draw a line parallel to another by a point given.

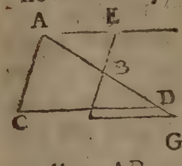


LET it be requir'd to draw a line by the point C, which ſhall be parallel to the line AB. Draw the line CE, and make the angle ECD equal to the angle CEA. I ſay the line CD is parallel to AB.

Demonſtration.

The alternate angles DCE and CEA are equal: therefore the lines CD and AB are parallels.

'The eleventh Maxim, i. e. *If a line falling* '*upon two others make the internal angles leſs than* '*two right angles, thoſe lines will concur,* may alſo 'now be eaſily demonſtrated.



Let the line AC, falling upon the lines AB and CD, make the internal angles ACD, and CAB, leſs than two right angles: I ſay that the lines AB and CD will concur. Let the angles ACD and CAE be equal to two right angles: the lines AE and CD will be parallels (*by*

(*by the* 28.) Take the line AB as long as you pleaſe, and by the point B draw EF parallel to CA. Then take the line EB ſo oft as it is neceſſary, to make it reach lower than the line CD; as in the preſent figure I have taken it only twice; ſo that EB and BF are equal. By the point F draw a parallel FG equal to AE, and joyn the line GB. I ſay that the line ABG is only one line; and that therefore the line AB concurring in FG, if the line CD be continu'd, ſince it cannot cut its parallel FG, it will cut the line BG between B and G.

Demonſtration.

The triangles AEB and EFG have the ſides AEF and FG, BE and BF, equal; as alſo the alternate angles AEB, and BFG, (*by the* 29.) therefore they are equal in all reſpects, (*by the* 4.) And the oppoſite angles ABE, and FBG, are equal; and by conſequence [*by the coroll. of the* 15] AB and BG make but one right line.

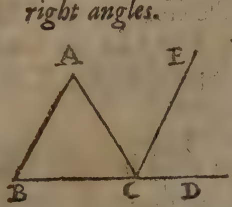
The USE.

'The uſe of parallel lines is very common; 'as in *Perſpectives*, for as much as the appear-'ances or images of lines parallel to the picture 'or table, are parallel among themſelves. In '*Navigation*, the lines of the ſame Rhomb of 'the wind are deſcrib'd by Parallels. *Polar* '*Dials* have the hour lines Parallels. The Com-'paſs of Proportion is founded alſo upon Paral-'lels.

PROPOSITION XXXII.

A THEOREM.

The external angle of a triangle is equal to both the internal opposite angles taken together; and all the three angles of a triangle are equal to two right angles.



LET the side BC of the triangle ABC be produc'd to D. I say, that the external angle ACD is equal to both the internal angles A and B taken together. By the point C draw the line CE parallel to the line AB.

Demonstration.

The lines AB and CE are parallels, therefore [*by the* 29.] the alternate angles ECA and CAB are equal; and [*by the same*] the external angle ECD is equal to the internal B. And by consequence the whole angle ACD, being equal to both the angles ACE, and ECD, of which it is compos'd, will be equal to both the angles A and B taken together.

In the second place. The angles ACD and ACB are equal to two right angles, (*by the* 13.) and I have demonstrated the angle ACD to be equal to both the angles A and B taken together;

ther; therefore the angles ACB, A, and B, that is to ſay, all the angles of the triangle ABC, are equal to two right angles, or which is all one, to 180 degrees.

Corollary 1. All the three angles of one triangle are equal to all the three angles of another triangle.

Coroll. 2: If two angles of one triangle be equal to two angles of another triangle, their third angles are alſo equal.

Coroll. 3. If a triangle has one right angle, the other two will be acute; and taken together will be equal to one right angle.

Coroll. 4. From a point given only one perpendicular can be drawn to the ſame line; becauſe a triangle cannot have two right angles.

Coroll. 5. A perpendicular is the ſhorteſt of all the lines, that can be drawn from the ſame point to the ſame line.

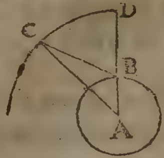
Coroll. 6. In a rectangle triangle the right angle is the greateſt angle, and the ſide oppos'd to it the greateſt ſide.

Coroll. 7. Every angle of an equilateral triangle contains 60 degrees; that is to ſay, the third part of 180.

The USE.

'This Propoſition is of uſe in *Aſtronomy*, to
' determine the *Parallax*. Suppoſe the point

'A to be the center of the earth; and that from



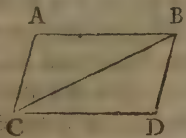
'the point B upon the ſuperfi-
'cies be taken the angle DBC,
'that is to ſay, the diſtance of a
'ſtar from the *Zenith* D. If the
'earth was tranſparent the ſtar
'would appear remote from the
'*Zenith* D, according to the bigneſs of the an-
'gle CAD, which is leſs than the angle CBD.
'For the angle CBD being an external angle
'in reſpect of the triangle ABC, it is (*by the*
'32.) equal to both the oppoſite angles A and
'C. Therefore the angle C will be equal to
'the exceſs of the angle CBD above the angle
'A. Whence I infer, that if I can know by
'the *Aſtronomical tables* how far remote from
'the *Zenith* the ſtar ought to appear to him
'that ſhould be at the center of the earth, and
'obſerve it at the ſame time *from the ſuperficies*,
'the difference of thoſe two angles will be the
'*Parallax* BCA.

PROP.

PROPOSITION. XXXIII.

A THEOREM.

Two lines drawn towards the same parts, from the extremity of two other lines that are equal and parallel, are also themselves equal and parallel.



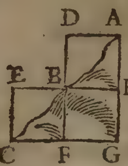
LET the lines AB and CD be parallel and equal; and let the lines AC and BD be drawn from their extremities towards the same parts; I say, that the lines AC and BD are equal and parallel. Draw the Diagonal line BC.

Demonstration.

Since the lines AB and CD are parallel, the alternate angles ABC and BCD will be equal; (*by the* 29) therefore in the triangles ABC and BCD, which have the side BC common, and the sides AB and CD equal, together with the angles ABC and BCD equal also, the bases AC and BD will be equal, (*by the* 4) and also the angles DBC, and BCA; which being alternate angles, the lines AC and BD will be parallel, (*by the* 27)

The USE.

' This Proposition is reduc'd to practice for
' the



'the measuring the perpendicular hights, AG, of the vastest 'Mountains; and also their horizontal lines, CG, which are hid 'by their bulk. Take a large square 'ADB, and place it so at the point 'A, that the side DB may fall perpendicularly; then measure the sides AD 'and DB. This done, do the same again at the 'point B, and measure BE and EC: the sides 'parallel to the horizon, AD, BE, added together give the horizontal line CG; and the 'perpendicular sides DB and EC, give the perpendicular hight AG. This way of measuring is called * *Cultellation.*

* *Measuring by piece-meal.*

PROPOSITION XXXIV.

A Theorem.

The opposite sides and angles of a Parallelogram are equal; and the diameter divides it into two equal parts.

SUppose the figure ABDC [*see the fig. of the preceding Prop.*] to be a Parellelogram, that is to say, that the sides AB, CD; AC, and BD, are parallel. I say, the opposite sides AB, CD; AC, and BD, are equal; as also that the angles

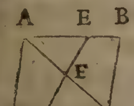
A

A and D, ABD, and ACD; and that the diameter BC equally divides the whole figure.

Demonstration.

The lines AB, and CD, are suppos'd to be parallels: therefore the alternate angles ABC and BCD will be equal, (*by the* 29.) In like manner the sides AC and BD being suppos'd to be parallels, the alternate angles ACB and CBD will be equal. And further, the triangles ABC, BCD, having the same side BC; and the angles ABC, BCD; ACB, and CBD equal, will be equal in all respects, (*by the* 26.) Therefore the sides AB, CD; AC, and BD, and the angles A and D, are equal: and the diameter divides the figure into two equal parts. And since the angles ABC, BCD; ACB, and CBD, are equal, joyning together ABC and CBD; and likewise BCD and ACB, we infer that the opposite angles ABD, and ACD are equal.

The USE.



‘Surveyors have need of this Proposition for dividing grounds. If the field be a Parallelogram, they can divide it into two equal parts by the diameter AD. But if you be oblig'd to divide it by the point E: divide first the diameter into two equal parts by the point F, then draw the line EFG, which will divide the figure into two equal parts. For

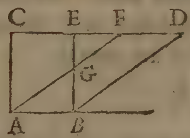
‘the

'the triangles AEF, and GFD, having the alter-
'ternate angles EAF, FDG; and AEF, FGD;
'and the ſides AF and FD equal, are equal,
'(*by the* 26) And ſince the Trapezium BEFD
'with the triangle AEF; that is to ſay, the tri-
'angle ADB, is half the parallelogram, (*by the*
'34.) the ſame Trapezium BEFD with the tri-
'angle DGF will be half the ſame. Therefore
'the line EG divides it in the middle.

PROPOSITION XXXV.

A THEOREM.

Parallelograms, having the ſame baſe, and being between the ſame parallels, are equal.



LET the Parallelograms be ABEC, and ABDF, having the ſame baſe AB, and being between the ſame parallels AB and CD. I ſay, they are equal.

Demonſtration.

The ſides AB, CE are equal, (*by the* 34) as alſo AB, FD: therefore CE and FD are equal; and adding to them EF, the lines CF and ED will be equal. The triangles therefore CFA, and EDB, have the ſides CA, EB, as alſo CF, and ED equal, together with the angles DEB, and FCA, (*by the* 29.) one being an external, and the

the other an internal angle on the ſame ſide. Therefore (*by the* 4.) the triangle ACF and BED are equal; and taking from them both, that which is common, *viz.* the little triangle EGF, the Trapezium FGBD will be equal to the Trapezium CAGE: and adding to both the triangle AGB, the Parallelograms ABEC and ABDF will be equal.

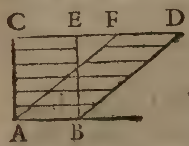
The U S E.

'*Scotus*, and ſome Divines ſince him, have 'made uſe of this Propoſition to prove, that '*Angels may extend themſelves to what ſpace they* '*pleaſe.* For ſuppoſing they can aſſume any 'figure, provided [they have not a greater ex-'tenſion: it is evident, that if an Angel ſhould 'poſſeſs the ſpace of the Parallelogram ABEC, 'it may likewiſe occupy the ſpace of the Paral-'lelogram ABDF; and becauſe parallels may 'be continu'd *in infinitum*, (without end,) and 'Parallelograms may be ſtill form'd longer and 'longer, which will all be equal to ABEC; an 'Angel will be able to extend it ſelf ſtill farther 'and farther.

A Demonſtration of the ſame Propoſition by Indiviſibles.

'This method was lately invented by *Cava-*'*lerius*; which has found different acception in the

'the world, ſome approving, and others rejecting it. His method conſiſts in this; that we imagine ſuperficies's to be compos'd of lines, like ſo many threds. And tis certain, that two pieces of linnen will be equal, if they have both the ſame number of threds, of equal length, and equally compacted.



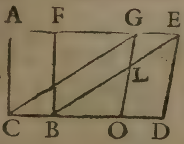
'Let two Parallelograms therefore ABEC, and ABDF, be propos'd, having the ſame baſe AB, and being between the ſame parallels AB, CD. Divide the Parallelogram ABEC into as many lines as you pleaſe, parallel to AB, which continue to the other Parallelogram ABDF. 'Tis evident there will be no more in one, than in the other; and that they will be of equal length, being all equal to the baſe AB; and that they will not be more cloſely compacted in one, than in the other: therefore the Parallelograms will be equal.

PROP.

PROPOSITION XXXVI.

A THEOREM.

Parallelograms, upon equal bases, and between the same parallels are equal.



LET the bases CB and OD of the parallelograms ACBF, ODEG, be equal; and let both be between the same parallels AE, CD. I say the parallelograms are equal. Draw the lines CG, and BE.

Demonstration.

The bases CB, and OD, are equal: OD, and GE, are also equal: therefore CB and GE are equal, and parallel; and by consequence (*according to the* 33.) CG and BE will be equal and parallel; and CBEG will be a parallelogram equal to CBFA, (*by the* 35.) having both the same base. In like manner, taking GE for the base, the parallelograms GODE and CBEG will be equal, (*by the same.*) Therefore the parallelograms ACBF, and ODEG, are equal.

The USE.

'We oft reduce parellelograms, which have 'oblique angles, as CBEG, or ODEG, to re-'ctangles; as CBFA: so that measuring the

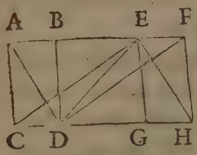
'latter,

'latter, which is easie, being only to muliply
'AC by CB, the product being equal to the
'Parallelogram ACBF, we may by consequence
'know the other Parallelograms CBEG, or
'ODEG.

PROPOSITION XXXVII.

A Theorem.

Triangles having the same base, and being between the same parallels, are equal.



IF the triangles ACD and CDE, have the same base CD, and be inclos'd between the same parallels AF, and CH, they will be equal. Draw the lines DB, and DF, parallel to the lines AC, and CE, and you will have form'd two Parallelograms.

Demonstration.

The Parallelograms ACDB, and ECDF, are equal (*by the* 35.) and the triangles ACD, CDE, are the halfs of those Parallelograms (*by the* 34.) Therefore the triangles ACD, CDE, are equal.

PROP.

PROPOSITION XXXVIII.

A THEOREM.

Triangles, that have equal bases, and are inclos'd within the same parallels, are equal.

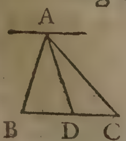
IF the triangles ACD, and EGH, [*see fig. preced.*] have equal Bases CD, and GH, and are inclos'd within the same parallels AF, and CH, they are equal. Draw the lines BD and HF parallel to the sides AC, and EG; and you will have form'd two parallelograms.

Demonstration.

The Parallelograms ACDB, and EGHF, are equal; (*by the* 36.) and the triangles ACD and EGH are the halfs of those parallelograms, (*by the* 34.) therefore they are also equal.

The USE.

'We have in these propositions directions for 'dividing a triangular field into two equal parts; 'for example the triangle ABC. 'Divide the line which you will 'take for the base, as BC, into two 'equal parts in D: I say the trian-'gles ABD, and ADC, are equal. 'For if you suppose a line drawn by A, parallel 'to BC, those triangles will have equal bases,

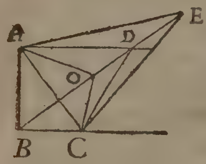


'and be inclos'd within the ſame parallels, and 'by conſequence will be equal. Other Diviſi-'ons, grounded upon the ſame propoſition, 'might be made; but I omit them, that I 'might not be tedious.

PROPOSITION XXXIX.

A THEOREM.

Equal trinagles, upon the ſame baſe, are within the ſame parallels.



IF the triangles ABC, and DBC, having the ſame baſe BC, be equal; the line AD drawn by the tops will be parallel to the baſe. For if AD and BC be no parallel; if you draw a paralled by the point A, it will fall either below the line AD, as AO; or above it, as AE. Suppoſe it to fall above, and produce BD till it meet the line AE, at the point E; then draw the line CE.

Demonſtration.

The triangles ABC and EBC are equal, (*by the* 38.) ſince the lines AE and BC are parallel; 'tis likewiſe ſuppos'd that the tirangles ABC, and BDC, are equal: therefore the triangles

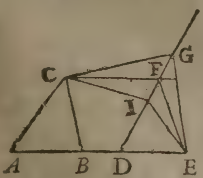
angles DBC and EBC would be equal; which is impoſſible, the firſt being part of the ſecond. Whence I conclude, that a line parallel to BC cannot be drawn above AD, as AE.

I add, that that parallel cannot be below AD, as AO: becauſe the triangle BOC would be equal to the triangle ABC, and by conſequence to the triangle DBC; that is to ſay, the part would be equal to the whole. It muſt therefore be confeſs'd, that the line AD is parallel to the lineBC.

PROPOSITION. XL.

A Theorem.

Equal triangles, having equal baſes, if they be taken upon the ſame line, are between the ſame parallels.



If the equal triangles ABC and DEF, have equal baſes AB and DE, taken upon the ſame line AE; the line CF drawn by their tops will be parallel to AE. For if it be not parallel, having drawn by the point C a line parallel to AE, it will paſs either above CF, as CG; or below it, as CI.

Demonſtration.

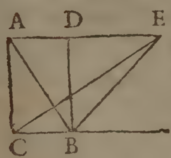
If it paſs above CF, as CG, continue DF till

it meet with CG in G; and draw the line EG. The triangles ABC and DGE would be equal, (*by the* 38.) and ABC and DEF being ſuppos'd to be equal, DEF, DGE, would be alſo equal; which, one being part of the other, cannot poſſibly be: therefore the parallel cannot paſs above CF. I add, that neither can it paſs below it, as CI; becauſe then the triangles ABG and DEI would be equal, and by conſequence DEI, and DEF; the part and the whole. Therefore only CF can be parallel to AE.

PROPOSITION XLI.

A THEOREM.

A Parallelogram will be double to a triangle, if they be between the ſame parallels, and have equal baſes.



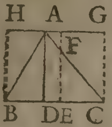
IF the Parallelogram ACBD, and the triangle EBC, be between the ſame Parallels AE and BC; and have the ſame baſe BC, or only equal baſes; the Parallelogram will be double the triangle. Draw the line AC.

Demonſtration.

The triangles ABC and BCE are equal, (*by the*

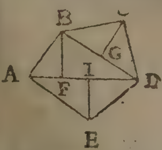
the 37.) But the Parallelogram ACBD is double the triangle ABC, (*by the* 34.) It is therefore double the triangle BCE. It would be alſo double a triangle, that, having a baſe equal to BC, ſhould be between the ſame parallels.

The USE.



'The ordinary method of meaſuring the *area* or ſuperficies of a triangle is built upon this Propoſition. If the triangle ABC be propos'd; from the angle A we muſt draw AD perpendicular to the baſe BC; then multiplying the perpendicular AD by half the baſe BE, the product gives the *area* of the triangle: becauſe multiplying AD, or what is the ſame, EF by BE, we have a rectangle BEFH, which is equal to the triangle ABC. For (*by the* 41.) the triangle ABC is half the rectangle HBCG; and ſo likewiſe is the rectangle BEFH.

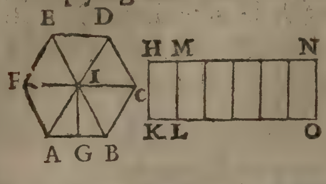
'We meaſure all ſorts of rectilineal figures,



'as ABCDE, by dividing them into triangles, as BCD, ABD, AED; drawing the lines AD, and BD; and the perpendiculars CG, BF, and EI. For multiplying half of BD by CG, and half of AD by BF, and by EI, we have the *area* of all thoſe triangles: adding which together the ſumm is equal to the rectilineal figure ABCDE.

 'We

'We find the *area* of the regular Polygones, by 'multiplying half their circuit by a perpendi-'cular drawn 'from their cen-'ter to one of 'their sides. For 'multiplying A 'G by IG, we



'shall have a rectangle HKLM equal to the 'triangle AIB: and repeating the same for all 'the other triangles, taking always half of the 'bases, we shall have a rectangle HKON, which 'will have the side KO compounded of all the 'half bases, and by consequence equal to half 'the circumference; and the side HK equal to 'the perpendicular IG.

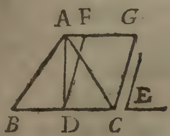
'Tis according to this principle, that *Archi-'medes* has demonstrated, that a circle is equal 'to a rectangle compris'd under the semidia-'meter, and a line equal to half the circumfe-'rence.

PROP-

PROPOSITION XLII.

A Problem.

To make a Parallelogram equal to a triangle given, having one angle eqaul to an angle given.



LET a Parallelogram be desir'd, equal to the triangle ABC, and having an angle equal to the angle E, divide the base BC into two equal parts at the point D; and draw the line AG parallel to BC, *(by the* 31.) then make the angle CDF equal to E, (*by the* 23.) and lastly draw the line CG parallel to DF: the figure FDCG is a Parallelogram, because the lines FG, DC; FD, and GC, are parallels, and its angle CDF is equal to the angle E; and farther, tis also equal to the triangle ABC.

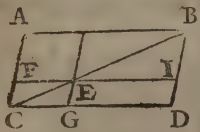
Demonstration.

The triangle ADC is half the parallelogram FDCG, (*by the* 41.) 'tis also half the triangle ABC; since the triangles ADC, and ADB, are equal, (*by the* 38.) Therefore the triangle AB C is equal to the Parallelogram FDCG.

PROPOSITION XLIII.

A THEOREM.

The complements of a parallelogram are equal.



In the Parallelogram ABDC, the complements AFEH, and EGDI, are equal,

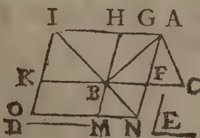
Demonstration.

The triangles ABC, and BCD, are equal, (*by the* 34.) therefore if the triangles HBE, and BIE; FEC, and CGE, which are also equal, (*by the same*,) be substracted, the complements AFEH, GDIE, which remain, will be equal.

PROPOSITION XLIV.

A PROBLEM.

To describe a parallelogram upon a line given, which shall be equal to a triangle, and have such a certain angle; i.e. equal to one given.



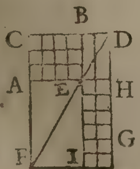
SUppose you be requir'd to make a parallelogram, which shall have one of its angles equal to the angle E, and one of its sides equal to the

the line D, and be equal to the triangle ABC. Make the Parallelogram BFGH, (*by the* 42.) which has the angle BFG equal to the angle E, and is equal to the triangle ABC. Produce the ſides GF, and GH, ſo that HI may be equal to the line D; and draw the line IBN till it cuts GF produc'd to N; and from the point N draw the line NO parallel to GI, and IO parallel to BH; producing alſo the ſide FB to K, and HB to M. The parallelogram MK is that which you deſire.

Demonſtration.

GF and HM being parallels, the alternate angles GFB or the angle E, and FBM, are equal (*by the* 29.) In like manner the lines KB and MN being parallel, the alternate angles FBM, and BMO, are equal; therefore the angle BMO is equal to E, and the ſide KB is equal to the line HI or D: and laſtly, the Parallelogram MK is equal to the Parallelogram GFBH, (*by the preceding*,) and that was made equal to the triangle ABC. Therefore the Parallelogram MK is equal to the triangle ABC.

The USE.



This Propoſition contains a kind 'of *Geometrical* Diviſion: for in '*Arithmetical* Diviſion a number 'is propos'd, which may be lookt 'on as a rectangle: for example, 'the rectangle AB. conſiſting of twelve

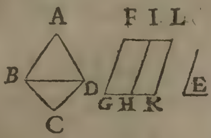
'twelve ſquare feet, which is to be divided by 'another number, ſuppoſe, two; that is to ſay, 'another rectangle is deſir'd to be made equal 'to AB, having one of its ſides, BD equal to 'two; and the queſtion is, how many feet the 'other ſide ought to contain; which is, as it were, 'the quotient. This is done Geometrically by 'the Rule and Compaſs. Take BD conſiſting 'of two feet, and draw the Diagonal DEF: 'the line AF is that which you ſeek. For hav-'ing compleated the rectangle DCFG, the com-'plements EG, and EC, are equal, (*by the* 43.) 'and EG has for one of its ſides EH, equal to 'BD, of two feet in length; and EI equal to 'AF. This kind of Diviſion, is call'd *Appli-'cation*, becauſe the rectangle AB is apply'd to 'the line BD, or EH: and from hence tis, that 'all Diviſion is frequently call'd *Application*; 'becauſe the ancient *Geometricians* made more 'uſe of the Rule and Compaſs, than of *Arith-'metick*.

PROP-

PROPOSITION XLV.

A PROBLEM.

To describe a parallelogram, which ſhall have a certain angle; and be equal to a rectilineal figure given.



LET the rectilineal figure propos'd be AB CD, to which you are required to make an equal Parallelogram, which ſhall have an angle equal to the angle E. Divide the rectilineal into triangles by the line BD: and (*by the* 42.) make a Parallelogram FGHI, which has the angle IHG equal to the angle E, and is equal to the triangle ABD; and (*by the* 44) make the Parallelogram IHKL equal to the triangle BCD, having one ſide equal to IH, and the angle LIH equal to the angle E. The Parallelogram FGKL will be equal to the rectilineal ABCD.

Demonſtration.

Nothing need be prov'd, but that the Parallelograms FGHI, and HKLI, make up but one; that is to ſay, GH, and HK, make but one right line. The angles GHI, and LKH, are equal to the angle E. And the angles LKH, and

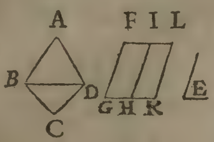
'twelve ſquare feet, which is to be divided by 'another number, ſuppoſe, two; that is to ſay, 'another rectangle is deſir'd to be made equal 'to AB, having one of its ſides, BD equal to 'two; and the queſtion is, how many feet the 'other ſide ought to contain; which is, as it were, 'the quotient. This is done Geometrically by 'the Rule and Compaſs. Take BD conſiſting 'of two feet, and draw the Diagonal DEF: 'the line AF is that which you ſeek. For hav-'ing compleated the rectangle DCFG, the com-'plements EG, and EC, are equal, (*by the* 43.) 'and EG has for one of its ſides EH, equal to 'BD, of two feet in length; and EI equal to 'AF. This kind of Diviſion, is call'd *Appli-'cation*, becauſe the rectangle AB is apply'd to 'the line BD, or EH: and from hence tis, that 'all Diviſion is frequently call'd *Application*; 'becauſe the ancient *Geometricians* made more 'uſe of the Rule and Compaſs, than of *Arith-'metick*.

PROP-

PROPOSITION XLV.

A PROBLEM.

To describe a parallelogram, which shall have a certain angle; and be equal to a rectilineal figure given.



LET the rectilineal figure propos'd be AB CD, to which you are required to make an equal Parallelogram, which shall have an angle equal to the angle E. Divide the rectilineal into triangles by the line BD: and (*by the* 42.) make a Parallelogram FGHI, which has the angle IHG equal to the angle E, and is equal to the triangle ABD; and (*by the* 44) make the Parallelogram IHKL equal to the triangle BCD, having one side equal to IH, and the angle LIH equal to the angle E. The Parallelogram FGKL will be equal to the rectilineal ABCD.

Demonstration.

Nothing need be prov'd, but that the Parallelograms FGHI, and HKLI, make up but one; that is to say, GH, and HK, make but one right line. The angles GHI, and LKH, are equal to the angle E. And the angles LKH, and

and KHI, are equal to two right angles, becauſe KHIL is a Parallelogram. Therefore the angles GHI and KHI are equal to two right angles, and (*by the* 14.) GH and HK make one right line.

The U S E.

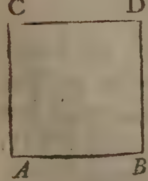
'The uſe of this propoſition is the ſame with 'the preceding; ſerving to meaſure the capa-'city of any figure whatſoever, by reducing it 'into triangles, and then making a rectangular 'Parallelogram equal to them.

'Tis eaſie likewiſe to make a rectangular Pa-'rallelogram upon a determinate ſide, which 'may be equal to many irregular figures. In 'like manner having many figures a rectangle 'may be deſcribed equal to their difference.

PROPOSITION. XLVI.

A PROBLEM.

To deſcribe a ſquare upon a line given.



TO deſcribe a ſquare upon the line AB, draw two perpendiculars AC and BD equal to AB, and draw the line CD.

Demonſtration.

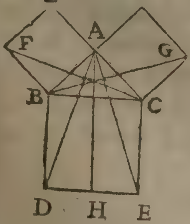
The angles A and B being right angles;

angles, the lines AC and BD are parallels (*by the* 28.) They are alſo equal; therefore the lines AB and CD are parallels and equals, [*by the* 33.] and the angles A and C equal to two right angles; as alſo B and D, [*by the* 29.] and ſince A and B are both right angles, the angles C and D will be ſo likewiſe. Therefore the figure AD has all its ſides equal, and all its angles right angles, and by conſequence is a ſquare.

PROPOSITION XLVII.

A THEOREM.

The ſquare of the baſe of a rectangular triangle, is equal to the ſquares of both the other ſides taken together.



SUppoſe the angle BAC to be a right angle, and that ſquares were deſcribed upon all the ſides BC, AB, and AC: that of the baſe BC, which is oppos'd to the right angle, will be equal to the ſquares or both the ſides AB, and AC. Draw the line AH parallel to BD, and CE; and joyn the lines AD, AE, FC, and BG. I will prove the ſquare AF is equal to the rectangle BH, and the ſquare AG to the rectan-

gel

gle CH; and therefore the ſquare BE is equal to both the ſquares AF and AG.

Demonſtration.

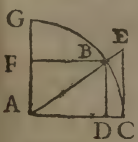
The triangles FBC, and ABD, have the ſides AB, BF; BD, and BC, equal: and the angles FBC, and ABD, are equal, each containing the angle ABC more than their reſpective right angles. Therefore [*by the* 4.] the triangles ABD and FBC are equal. But the ſquare AF is double the triangle FBC, [*by the* 41.] having the ſame baſe BF, and being between the ſame parallels BF, and AC. In like manner the rectangle BH is double the triangle ABD, having likewiſe the ſame baſe BD, and being between the ſame parallels BD and AH. Therefore the ſquare AF is equal to the triangle BH. By the ſame method the triangles ACE, and GCB; may be prov'd to be equal, [*by the* 4.] and the ſquare AG to be double the triangle GCB; and the rectangle CH, double the triangle ACE, [*by the* 41.] therefore the ſquare AG is equal to the rectangle CH; and by conſequence the ſquares AF and AG are equal to the ſquare BDEC.

The USE.

'''Tis ſaid that *Pythagoras*, having found out 'this Propoſition, ſacrific'd a *Hecatomb*, i. e. a 'hundred Oxen, to the *Muſes*, to return them 'thanks for their aſſiſtance; ſuppoſing it, it

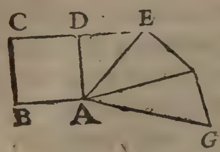
ſeems

‘ seems, above the power of bare humane in-
‘ vention. Nor was his esteem thereof so irra-
‘ tional, as to some perhaps it may appear; this
‘ Proposition being the foundation of a very con-
‘ siderable part of the *Mathematicks.* For in
‘ the first place *Trigonometry* cannot possibly
‘ subsist without it, it being necessary to com-
‘ pose a table of all the lines that may be in-
‘ scrib'd in a circle, as Chords, Sines, Tangents,
‘ Secants; as may appear by one example.



‘ Suppose the semidiameter AC to
‘ be divided into 100000 parts, and
‘ that the arch EC contains 30 de-
‘ grees. Since the Chord, or line
‘ that subtends 60 degrees is equal
‘ to the semidiameter AC; BD the
‘ sine of 30 degrees, will be equal to half AC,
‘ and therefore contain 50000 parts. Now in
‘ the rectangular triangle ADB, the square of
‘ AB is equal to the squares of AD, and BD.
‘ Make therefore the square of AB, by multi-
‘ plying 100000 by 100000, and from the pro-
‘ duct substract the square of 50000 or BD;
‘ the remainder will be the square of AD, or
‘ BF the sine of the complement: and extract-
‘ ing the square root of that number, you will
‘ have the line FB. This done, making as AD
‘ to BD, so AC to CE, you will have the tangent
‘ CE; then adding together the squares of
‘ AC and CE, the product [*by the* 47.] will give
the

'the ſquare of AE; extracting therefore from 'that number the ſquare root, you will know 'the length of the line AE, which is the ſe-'cant.



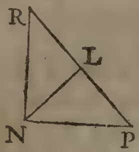
'By this alſo we may aug-'ment figures, as much as 'we pleaſe. For example; 'to double the ſquare ABC 'D, continue the ſide CD, 'ſo that AD and DE may 'be equal: the ſquare of 'AE will be double the ſquare of ABCD; ſince '(*by the* 47.) it is equal to both the ſquares of 'AD and DE. Making the right angle AEF, 'and taking EF equal to AB, the ſquare of AF 'will be triple the ſquare ABCD. Again, 'making the right angle AFG, and taking FG 'equal to AB, the ſquare of AG will be qua-'druple, or four times the ſquare of ABCD. 'And that which I ſay of the ſquare, may be 'underſtood of all ſimilar figures.

PROP.

PROPOSITION. XLVIII.

A THEOREM.

If in a triangle the ſquare of one ſide be equal to the ſquares of both the other ſides, taken together; the angle oppoſite to that firſt ſide will be a right angle.



IF the ſquare of the ſide NP be equal to both the ſquares of the ſides NL, and LP, taken together; the angle NLP will be a right angle. Draw LR perpendicular to NL, and equal to LP; then draw the line NR.

Demonſtration.

In the rectangular triangle NLR, the ſquare of NR is equal to the ſquares NL, and RL, or LP, (*by the* 47.) Now the ſquare of NP is alſo equal to the ſame ſquares of NL, and LP; therefore the ſquare of NR is equal to the ſquare of NP, and by conſequence the lines NR and NP are equal. And becauſe the triangles NLR, and NLP, have the ſide NL common; the ſides LP and LR equal, and their baſes NP and NR alſo equal; the angles NLP and NLR will be equal, (*by the* 8.) and the angle NLR being a right angle, the angle NLP muſt ſo too.

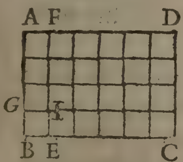
THE SECOND BOOK OF THE ELEMENTS OF EUCLID.

'EUCLID in this Book treats of the
'powers of right Lines; that is to ſay,
'of their Squares; comparing the di-
'vers Rectangles, which are made upon a Line
'divided, as well with the Square, as the
'Rectangle, of the whole line. 'Tis a part
'exceeding uſeful, ſerving for the foundation
'of the principal Operations of *Algebra*. The
'three firſt Propoſitions demonſtrate the third
'Rule, or Operation of *Arithmetick*, *Multi-
'plication*. The fourth teaches to extract the
'ſquare Root of any number whatſoever. Thoſe
'that follow to the Eighth ſerve upon many oc-
'caſions in *Algebra*. The reſt inſtruct us in O-
'perations proper for *Trigonometry*. This Book
'ſeems at firſt view very difficult; becauſe
'men are apt to imagine there is ſomething my-
'ſterious contain'd therein; nevertheleſs the
'great-

‘ greateſt part of its Demonſtrations are ground-
‘ ed on this moſt evident Principle, That the
‘ whole is equal to all its parts taken together.
‘ But it ought not to diſcourage any, if they
‘ ſhould not at the firſt attempt fully compre-
‘ hend them.

DEFINITIONS.

1. *A rectangular parallelogram is compris'd under two lines, that form a right angle.*



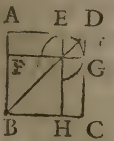
‘ OBſerve that henceforward by a rectangle we ſhall intend ſuch a parallelogram, whoſe angles are all right angles, diſtinguiſhing it by giving its longitude and latitude, naming two of its ſides, which contain one of its angles, as the lines AB and BC. For the rectangle ABCD is compris'd under the lines AB and BC; BC denoting its longitude, and AB its latitude; and the other being equal to theſe it will not be neceſſary to name them. I have alſo formerly intimated that the line AB, remaining perpendicular to BC, and being mov'd from one extremity thereof to the other, produces the rectangle ABCD; and that that

'motion has ſome reſemblance to *Arithmetical*
'multiplication: ſo that, as the line AB mov-
'ing over the line BC, that is, taken ſo many
'times, as there are points in BC, compoſes
'the rectangle ABCD: ſo the multiplication
'alſo of AB by BC, will give the rectangle
'ABCD. As, ſuppoſe I knew the number of
'*Mathematical points*, that are in AB, for
'example 40, and that there were 60 in BC:
'it is evident that the rectangle ABCD will
'have ſo many lines equal to AB, as there are
'points in BC; and that multiplying 40 by
'60, the product will be 2400, which is the
'number of *Mathematical points* in the rectan-
'gle ABCD.

'I may take what quantity I pleaſe for a *Ma-*
'*thematical point*; provided I do not afterwards
'ſubdivide it; it muſt therefore be obſerv'd;
'that when I meaſure a line, for a *Mathemati-*
'*cal point* I take that meaſure which beſt ſuits
'with my occaſions; for example, when I ſay
'a line of five foot in length, my *Mathema-*
'*tical point* is a line of a foot long; which I
'take without conſidering that it is compos'd
'of any parts. In meaſuring a ſuperficies like-
'wiſe I do the ſame, taking ſome known ſuper-
'ficies, for example, a foot ſquare; which I do
'not afterwards ſubdivide. I make uſe of a
'ſquare rather than any other figure; becauſe
'its length and bredth being equal there is no

'need

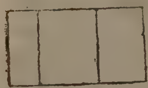
'need of naming more than one of its dimensi-
'ons to describe it. Accordingly when I would
'mark out the *Area* of the rectangle ABCD,
I do not consider the sides as simple lines, but
'as rectangles of a determinate bredth: for
'example, when I say that the rectangle ABCD
'has the side AB of four foot long, since a foot
'is to me instead of a *Mathematical point*, I
'conceive the side AB to have also a foot in
'bredth, and to be as the rectangle AbEF.
'Therefore knowing how many times the
'bredth BE is contain'd in the line BC, I
'shall know how many times the line AB is
'contain'd in the rectangle ABCD; that is to
'say, multiplying AB which has four foot
'square, by 6, the product will be 24 foot square.
'In like manner knowing the magnitude of the
'rectangle ABCD to be 24 foot square, and one
'of its sides AB to be 4; dividing 24 by 4
'the quotient will give me the other side BC,
'consisting of six foot square.



2. Having drawn the diameter of a rectangle, one of the lesser rectangles thro which it passes, together with the two complements, is call'd the Gnomon. 'As the rectangle EG, 'thro which the diameter BD passes,
'together with the complements EF and GH,
'is call'd the Gnomon; their figure together
'representing a Carpenters square.

PROPOSITION I.
A THEOREM.

If two lines be propos'd, whereof one is divided into divers parts, the rectangle contain'd under those two lines is equal to the rectangles contain'd under the line which is not divided, and the parts of the line divided.



LET the lines propos'd be AB, and AC; and let AB be divided into as many parts as you pleaſe. The rectangle AD contain'd under the lines AB and AC, is equal to the rectangle AG contain'd under AC and AE; to the rectangle EH contain'd under EG equal to AC, and EF; and to the rectangle FD contain'd under FH equal to AC and FB.

Demonſtration.

The rectangle AD is equal to all its parts taken together; which are the rectangles AG, EH, and FD; and no other. Therefore the rectangle AD is equal to the rectangles AG, EH and FD taken togerher.

By Numbers.

This propoſition holds true likewiſe in numbers. Suppoſe the line AC to be five foot long; AE two, EF four, FB three; and by conſequence AB nine: the rectangle contain'd under

under AC five; and AB nine, that is to say, five times nine, which makes forty five, is equal to twice five or ten, four times five or twenty, and three times five, or fifteen; for ten, twenty, and fifteen, make forty five.

The USE.

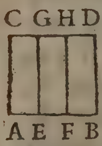
A,	53
B,	8
C,	50,3
B,	8
D,	24
E,	400
F,	424

‘By this proposition is demonstrated the ordinary operation of multiplication. For example, if you were to multiply the number A, which is 53, by the number B, that is 8. Divide the number A into so many parts as there are characters: that is, two, 50, and 3; which multiply by 8, saying, eight times three is twenty four; and so you make one rectangle. Then multiplying the number 50 by 8, the product will be 400. But 'tis evident that the product of eight times 53, being 424, is equal to the product of 24, and the product 400 taken together.

PROPOSITION II.

A Theorem.

The ſquare of any line is equal to the rectangles contain'd under the whole line, and all its parts.



LET the line propos'd be AB, and its ſquare ABCD. I ſay the ſquare ABCD is equal to the rect-angle contain'd under the whole line AB, and AE; another under AB and EF; and a third under AB and FB.

Demonſtration.

The ſquare ABCD is equal to all its parts taken together, which are the rectangles AG, EH, FD. The firſt AG is contain'd under AC equal to AB, and AE. The ſecond EH s contain'd under EG equal to AC or AB, and FE. The third FD is contain'd under FH e-qual to AB, and FB: and 'tis the ſame thing to be contain'd under a line equal to AB, and to be contain'd under AB it ſelf. Therefore the ſquare of AB is equal to the rectangles contain'd under AB, and AE, EF, FB, the parts of AB.

By Numbers.

Let the line AB repreſent the number nine: its

its ſquare will be 81: Let alſo the part AE be four; EF three, and EC two: nine times four make thirty ſix; nine times three twenty ſeven, and nine times two eighteen, and 'tis plain that 36, 27, and 18 make 81.

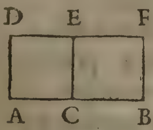
The U S E.

'This Propoſition ſerves likewiſe to prove 'multiplication; as alſo for Equations in *Al-* '*gebra.*

PROPOSITION III.

A THEOREM.

If a line be divided into two parts, the rectangle contain'd under the whole line, and one of its parts, is equal to the ſquare of the ſame part, and the rectangle contain'd under both the parts.



LET the line AB be divided into two parts at the point C; and let a rectangle be made of the whole AB, and one of its parts AC, that is to ſay, let AD be equal to AC; and then if the rectangle AF be compleated, it will be equal to the ſquare of AC, and the rectangle contain'd under AC and CB. Draw the perpendicular GE.

De-

Demonstration.

The rectangle AF contain'd under AB and AD equal to AC, is equal to all its parts, which are the rectangles AE, and CF. The first AE is the square of AC, the lines AC and AD being equal; and the rectangle CF is contain'd under CB, and CE equal to AD or AC. Therefore the rectangle contain'd under AB and AC is equal to the square of AC, and the rectangle contain'd under AC and CB.

By Numbers.

Let AB be 2; and AC 3; and CB 5: the rectangle contain'd under AB and AC, will be three times eight, or 24: the square of AC 3, is nine; and the rectangle contain'd under AC 3, and CB 5, is three times 5 or 15. But it is evident that 15 and 9 make 24.

The USE.

A,	43
C, 40,	3
B,	3
120.	9.
129.	

'The use of this Proposition is still to 'demonstrate the ordinary practise 'of multiplication. For example, if 'you would multiply the number 43 'by 3; having divided the number '43 into 40, and 3: three times 43 'will amount to as many as three times three, or 'nine, that is the square of three; and three 'times 40, that is, 120; for three times forty 'three is 129. Beginners ought not to be dis-

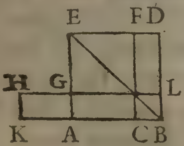
'cour-

'courag'd, if they do not presently apprehend 'these Propositions; which yet, in truth are 'not difficult, but as they are conceiv'd to con-'tain some strange mystery.

PROPOSITION IV.

A Theorem.

If a line be divided into two parts, the square of the whole line, will be equal to the squares of both the parts; and two rectangles contain'd under the same parts.



LET the line AB be divided in C, and its square ABDE describ'd; let its diagonal also EB be drawn and a perpendicular cutting it CF: and by that point let the line GL be drawn parallel to AB. 'Tis evident that the square ABDE, is equal to the four rectangles GF, CL, CG, and LF. The two first of which are the squares of AC and CB: and the two Complements are contain'd under AC and CB.

Demonstration.

The sides AE and AB are equal: therefore the angles AEB, and ABE are half right angles

angles: and becauſe the lines GL and AB are parallels; the angles of the triangles of the ſquare GF (*by the* 29. 1.) will be equal; as alſo their ſides (*by the* 6. 1.) Therefore GF is the ſquare of AC. In like manner CL is the ſquare of CB: the rectangle GC is contain'd under AC, and AG equal to BL or BC; and the rectangle LF is contain'd under LD equal to AC, and FD equal to BC.

Coroll. If you draw the diagonal of a ſquare the rectangles which it cuts are ſquares.

The U S E.

A,	144
B,	22
C,	12

'This Propoſition teaches the method of extracting the ſquare root of any number propos'd. Let the number be A, or 144, repreſented by the ſpuare AD, and its root by the line AB. I ſuppoſe it known from other principles that it requires two characters. I imagine therefore that the line AB is divided in C, ſo that AC may repreſent the firſt character, and BC the ſecond: Then ſearching the root of the firſt character of the numbe 144, which is 100, I find it to be 10: and making its ſquare 100 repreſented by the ſquare GF, I ſubſtract it from 144; and there remains 44 for the rectangles GC, FL; and the ſquare CL: But becauſe the figure of a Gnomon is not proper for this operation, I tranſport the rectangle FL unto KG, making one whole

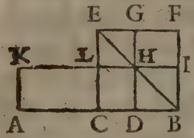
'rectangle KL, that is, 44. I know also already almost the whole side KB: for AC being 10, KC must be 20. I must therefore divide 44 by 20; that is to say, for my Divisor doubling the root found; I enquire then how many times 20 I can have in 44? and find twice; and therefore take 2 for the side BL; and because 20 was not the intire side KB, but only KC; that two which came in the quotient I add to Divisor, making it 22; which number being found precisely twice in 44, adding 2 to the root before found, I conclude the whole square root of 144 to be 12. You see then that the square 144 is equal to the square of 10, which is 100, the square of 2, that is 4; and twice 20, which makes the two rectangles contain'd under two and ten.

PROP-

PROPOSITION V.

A THEOREM.

If a line be divided into two equal parts and two parts that are unequal; the rectangle contain'd under the unequal parts, together with the ſquare of the intermediate part; is equal to the ſquare of half the line.



IF the line AB be divided into two equal parts in C; and two unequal parts in D; the rectangle AH contain'd under the unequal ſegments AD, and DB, with the ſquare CD, will be equal to the ſquare of CB, that is, the ſquare CF. Compleat the figure as you ſee; the rectangles LG and DI will be ſquares. (*by the coroll. of the* 4.) I will prove then that the rectangle AH, contained under AD, and DH equal to DB, with the ſquare LG, is equal to the ſquare CF.

Demonſtration.

The rectangle AL is equal to the rectangle DF, both being contain'd under half the line AB, and DB, or DH, which is equal to it. Add to both the rectangle CH; the rectangle AH will be equal to the Gnomon CBG. Add there-

therefore again to both the ſquare LG; and the rectangle AH, with the ſquare LG will be equal to the ſquare CF.

By Numbers.

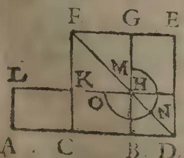
Let AB be 10; AC will be 5, and CB likewiſe; and let CD be 2, and DB 3. the rectangle contain'd under AD 7 and DB 3, that is to ſay 21, with the ſquare of CD 2, that is 4, will be equal to the ſquare CB 5, which is 25.

The USE.

'This Propoſition is very uſeful in the third 'Book: It is alſo us'd in *Algebra*, to demon- 'ſtrate the manner of finding the root of an af- 'fected or impure ſquare.

PROPOSITION VI.
A THEOREM.

If a line be divided into two equal parts, and to it another line added; the rectangle contain'd under the line compounded of thoſe two, and that which is added, with the ſquare of half the divided line, is equal to the ſquare of the line compounded of that half, and the line that is added.



IF to the line AB, divided into two equal parts in C, be added the line BD; the rectangle AN, contain'd under the line AD, and DN equal to BD, with the ſquare

of

of CB, is equal to the ſquare of CD. Make the ſquare of CD, and having drawn the diagonal FD, draw alſo BG parallel to EC, cutting FD at the point H, through which paſſes the line HN parallel to AD. KG will be the ſquare of CB; and BN, that of BD.

Demonſtration.

The rectangles AK, and CH, being upon equal baſes AC and CB, are equal(*by the* 36.1.) The complements CH and HE are equal, (*by the* 43. 1.) therefore the rectangles AK and HE are equal. Add to both the rectangle CN, and the ſquare KG: the rectangles AK and CN that is, the rectangle AN, with the ſquare KG, will be equal to the rectangles CN and HE, and the ſquare KG, that is, the ſquare CE.

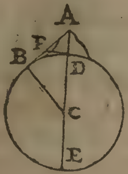
By Numbers.

Let AB conſiſt of 8 parts,: AC of 4; and CB of 4; BD of 3. ſo that the whole AD be 11. Tis evident the rectangle AN is three times 11, that is 33; which with the ſquare of KG, equal CB 4, that is 16; make 49, and therefore is equal to the ſquare of CD 7. which is 49; for 7 times 7 make 49.

The USE.

‘ *Maurylocus*, by the help of this Propoſition ‘ meaſur'd the whole Earth at one ſingle Obſervation.

vation. To effect which, he advises, that 'from the top of a mountain of 'known height A, you observe the 'angle BAC, made by the line 'AB, touching the superficies of 'the earth at B, and the line AC 'passing through the center: and 'that in the triangle ADF, know-



'ing the angle A, and the right 'angle ADF, you find by *Trigonometry* the sides 'AF and FD: and because tis easie to demon-'strate that FB and FD are equal, you will then 'know the line AB, and also its square. Now 'we have demonstrated in the preceding Pro-'position, that the line ED being divided into 'equal parts in C, and the line AD added to 'it; the rectangle contain'd under EA, and 'AD, with the square of CD, or CB, is equal 'to the square of CA; and the angle ABC, 'being a right angle, (as is prov'd in the third 'book) the square of CA is equal to the squares 'of AB and BC; therefore the rectangle un-'der AE and AD, with the square of BC, is 'equal to the squares of AB and BC. Take 'therefore from them both the square of BC, 'and the rectangle under AE, and AD, will be 'equal to the square of AB. Divide therefore 'the known square of AB, by the height of the 'mountain AD, and the Quotient will be the 'line AE; from which subtracting the hight

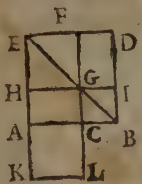
'of the mountain, the remainder will be the 'diameter of the earth DE.

'We have made uſe likewiſe of the ſame pro-'poſition in our *Algebra*, to demonſtrate the thir-'teenth propoſition of the third Book, to find 'the root of a ſquare equal to a more certain 'number of roots. The two that follow do 'alſo ſerve for the proof of the like operati-'ons.

PROPOSITION. VII.

A Theorem.

If a line be divided, the ſquare of the whole line, with that of one of its parts, is equal to two rectangles contain'd under the whole line, and that firſt part together with the ſquare of the other part.



LET the line AB be divided any where in C; the ſquare AD of the line AB, with the ſquare AL, will be equal to two right angles contain'd under AB and AC, with the ſquare of CB. Make the ſquare of AB, and having drawn the diagonal EB, and the lines CF and HGI; prolong EA ſo far, as that AK may be equal to AC: ſo AL will be the ſquare of AC, and HK will be equal to AB; For HA is equal to

to GC, and GC is equal to CB, becauſe CI is the ſquare of CB, (*by the Coroll. of the 4.*)

Demonſtration.

Tis evident, that the ſquares of AD and AL are equal to the rectangles HL and HD, and the ſquare CI. Now the rectangle HL is contain'd under HK equal to AB, and KL equal to AC. In like manner the rectangle HD is contain'd under HI equal to AB, and HE equal to AC. Therefore the ſquares of AB and AC are equal to two rectangles contain'd under AB and AC, and the ſquare of CB.

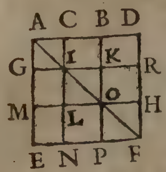
In Numbers.

Suppoſe the line AB to conſiſt of 9 parts, AC of 4, and BC of 5. The ſquare of AB 9 is 81, and that of AC 4 is 16; which 81 and 16 added together make 97. Now one rectangle under AB and AC, or 4 times 9, make 36, which taken twice is 72: and the ſquare of CB 5 is 25; which 72 and 25 added together make alſo 97.

PROPOSITION VIII.

A Theorem.

If you divide a line, and add another to it equal to one of its parts, the ſquare of the whole compounded line will be equal to four rectangles contain'd under the firſt line, and that part that is added together with the ſquare of the other part.



LET the line AB be divided any where in the point C, and BD equal to CB added to it: the ſquare of AD will be equal to four rectangles contain'd under AB, and BC or BD, and the ſquare of AC. Make the ſquare of AD, and having drawn the diagonal AE, draw likewiſe the perpendiculars BP, and CN, cutting the diagonal in I, and O: and alſo the lines MOH, and GIR, parallel to AB. The rectangles GC, LK, PH, MB, and NR, will be ſquares, (*by the Coroll. of the 4.*)

Demonſtration.

The ſquare ADEF is equal to all its parts; and the rectangles LB, OD, PM, are contain'd under lines equal to AB, and BC, and if you add the rectangle MI to the rectangle PH, they together will give you another rectangle contain'd under

under a line equal to AB, and another equal to CB or BD. Besides which there remains nothing but the square GC, which is the square of AC. Therefore the square AD is equal to four rectangles contain'd under AB and BD, and the square of AC.

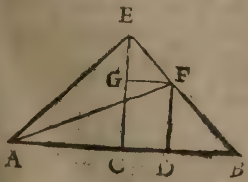
In Numbers.

Let the line AB consist of 7 parts, AC of 3, and CB of 4; as also BD: the square of AD 11 will be 121, And one rectangle under, AB 7 and BD 4, makes 28; which taken four times is 112; and those together with the square of 3, which is 9, make also 121.

PROPOSITION IX.

A Problem.

If a line be divided into two equal parts, and two unequal, the squares of the unequal parts will be double the square of half the line, and the square of the intermediate part.



Let the line be divided into two equal parts at the point C, and two unequal at the point D: the squares of the unequal parts A D, and DB will

will be double the ſquares of AC, which is half AB, and CD the intermediate part. Draw CE perpendicular to AB, and equal to AC; draw alſo the lines AE, and BE, and the perpendicular DF, as likewiſe FG parallel to CD. Then joyn the line AF.

Demonſtration.

The lines AC and CE are equal, and the angle C is a right angle: therefore (*by the* 5.1.) the angles CAE, and CEA, are equal; and conſequently half right angles. In like manner, the angles CEB, CBF, GFE, and DFB, are half right angles; and the line GF and HE, DF and DB, equal, (*by the* 6. 1.) and the whole angle AEF is a right angle. Now the ſquare of AE (*by the* 47. 1.) is equal to the ſquares of AC and CE, which are equal: therefore it is double the ſquare of AC. For the ſame reaſon, the ſquare of EF is double the ſquare of GF or CD. Now the ſquare of AF is equal to the ſquares of AE, and EF, becauſe the angle AEF is a right angle: therefore the ſquare of AF is double the ſquares of AC, and CD. The ſame ſquare of AF is likewiſe equal to the ſquares of AD, and DF or DB, the angle D being a right angle. Therefore the ſquares of AD, and DB, are double the ſquares of AC, and CD.

In Numbers.

Let AB be 10, AC 5, CD 3, and DB 2;

2: the ſquares of AD 8, and DB 2, that is to ſay, 64 and 4, which make 68, are double the ſquares of AC 5, that is, 25, and of CD 3, which is 9: for 25 and 9 make 34, which is half of 68.

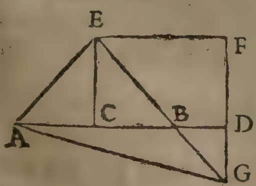
The USE.

"I have not met with this Propoſition, ex-
"cept in *Algebra*; no more than that which
"follows.

PROPOSITION X.

A THEOREM.

If a line be added to another that is divided into two equal parts; the ſquare of the line compounded of thoſe two, with the ſquare of that which is added, makes double the ſquare of half the line, and the ſquare of that which is compounded of half, and the line that is added.



LET the line AB be divided in the middle at the point C, and the line BD added to it: the ſquares of AD, and BD, will be double the ſquares of AC, and CD. Draw the perpendiculars CE and DF equal to AC: and then draw the lines

AE, EF; and producing FD to G, ſo that DG may be equal to BD, joyn the lines AG, and EBG.

Demonſtration.

The lines AC, CB, and CE being equal, and the angles at the point C being right angles: the angles CAE, AEC, CEB, and CBE, will be half right angles. In like manner the angle D being a right angle, and the lines BD and DG equal, the angles DBG, and DGB, will be half right angles; and ſo will likewiſe GEF, the angle F being a right angle; therefore the lines FG and FE are equal, (*by the* 6. 1.) and EF is equal to CD, (*by the* 33. 1.) Now the ſquare of AE is double the ſquare of AC, and the ſquare of EG alſo double the ſquare of EF, or CD, (*by the* 47. 1.)But the ſquare of AG is equal to the ſquares of AE and EG, (*by the ſame*:) therefore the ſquare of AG is double the ſquares of AC, and CD. The ſame ſquare of AG is likewiſe (*by the ſame*) equal to the ſquares of AD, and DG equal to DB: therefore the ſquares of AD and DB are double the ſquares of AC and CD.

By Numbers.

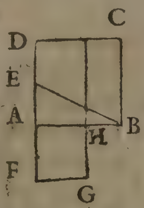
Let AB contain 6 parts, AC 3, and CB 3, BD 4; the ſquare of AD 10 is 100; the ſquare of BD 4 is 16, which make together 186. The ſquare alſo of AC 3 is 9: the ſquare

ſquare of CD 7 is 49. Now 49 and 9 make 58, the half of 116.

PROPOSITION XI.

A Problem.

To divide a line in ſuch a manner, that the rectangle under the whole line, and one of its parts, ſhall be equal to the ſquare of the other part.



Suppoſe the line AB to be divided in ſuch a manner, that the rectangle under the whole line AB, and BH, may be equal to the ſquare of AH. Make the ſquare of AB,(*by the* 46. 1.)and dividing AD in the middle in E, draw EB,and take EF equal to EB. Then make the ſquare of AF, that is to ſay, let AF and AH be equal. I ſay, the ſquare of AH will be equal to the rectangle HC, contain'd under HB, and BC equal to AB.

Demonſtration.

The line AD is divided equally in the point E, and the line FA added to it, therefore (*by the* 6.) the rectangle DG contain'd under DF, and FG equal to AF, with the ſquare of AE, is equal to the ſquare of EF,equal to EB: now the ſquare EB is equal to the ſquares of AE and

AB

AB, (*by the* 47.1.) therefore the ſquares of AB and AE are equal to the rectangle DG, and the ſquare of AE: and ſubtracting from both the ſquare of AE; the ſquare of AB, that is, AC, will be equal to the rectangle DG: taking away therefore the rectangle DH, which is common to both, the rectangle HC will be equal to the ſquare of AH, that is, AG.

The USE.

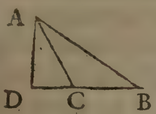
'This Propoſition teaches how to cut a line 'according to the extreme and middle propor'tion, as will be ſhewn in the *6th* Book, 'Tis 'alſo frequently made uſe of in the 14*th* Book 'of *Euclid*'s *Elements*, to find the ſides of regu'lar Solids. It is uſeful alſo in the 11. *of the* '4. to inſcribe a Pentagone in a circle, as alſo a 'Pentedecagone (or a figure with 15. angles.) 'You will ſee alſo other uſes thereof in divi'ding lines on this manner, in the 30*th* Pro'poſition of the 6.

PROP-

PROPOSITION XII.

A THEOREM.

In an Obtuſe triangle, the ſquare of the ſide oppos'd to the obtuſe angle, is equal to the ſquares of both the other ſides, and two rectangles contain'd under the line upon which a perpendicular will fall, and the line which lies betwixt the triangle and the perpendicular.



LET the angle ACB, of the triangle ABC, be an obtuſe, and let AD be drawn perpendicular to BC; the ſquare of the ſide AB is equal to the ſquares of the ſides AC and CB, and two rectangles contain'd under the ſide BC, and DC.

Demonſtration.

The ſquare of AB is equal to the ſquares of AD, and DB, (*by the* 47. 1.) But the ſquare of DB is equal to the ſquares DC, and CB, and two rectangles contain'd under DC and CB, (*by the* 4.) Therefore the ſquare of AB is equal to the ſquares AD, DC, and CB. and two rectangles contain'd under DC and CB. In ſtead of the two firſt ſquares AD, and DC, put the ſquare of AC, which is equal to them, (*by the* 47. (1 The ſquare AB will be equal to the ſquares AC

AC and CB, and two rectangles contain'd under DC and CB.

The USE.

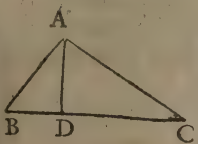
'This Propoſition is uſeful in *Trignometry* 'to meaſure the *area* of a triangle, whoſe ſides 'are known. For Example, ſuppoſe the ſide 'AB to conſiſt of twenty foot, AC of 13, BC 'of 11: the ſquare of AB will be 400, that of 'AC 169, and that of BC 121. The ſumm 'of the two laſt is 290, which ſubtracted from '400, there will remain 110 for the two rect-'angles under BC and CD. The half of which, '55, will make one half of thoſe rectangles; 'dividing which number by BC, 11, we ſhall 'have 5 for the line CD; whoſe ſquare 25 be-'ing ſubtracted from the ſquare of AC, 169, 'leaves the ſquare of AD, 144, whoſe root 12 'will be the ſide AD, which being multiplied 'by 5^2, the half of BC, will give the *area* of 'the triangle ABC, that is, 66 foot ſquare.

PROP.

PROPOSITION XIII.

A THEOREM.

In any triangle whatſoever, the ſquare of the ſide oppos'd to the acute angle, with two rectangles contain'd under the ſide upon which the perpendicular falls, and the line which is betwixt the perpendicular and that angle; is equal to the ſquares of both the other ſides.



Suppoſe the triangle to be ABC, and the acute angle C, and AD the perpendicular falling upon BC: the ſquare of the ſide AB, oppos'd to the acute angle C, with two rectangles contain'd under BC and CD, will be equal to the ſquares of AC, and BC.

Demonſtration.

The line BC being divided in D, (*by the* 7) the ſquares of BC and DC are equal to two rectangles under BC and CD, and the ſquare of BD. Add to both the ſquare of AD: the ſquares of BC, DC, and AD, will be equal to two rectangles under BC, and CD, and the ſquares of BD, and A D. Inſtead of the ſquares of CD, and AD, put the ſquare of AC, which is equal to them, (*by the* 47. 1.) and inſtead of the

the ſquares of BD, and AD, ſubſtitute the ſquare of AB, which is equal to them, (*by the ſame*:) the ſquares of BC, and AC, will be equal to the ſquare of AB, and two rectangles contain'd under BC, and CD

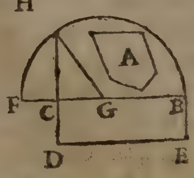
The USE.

'These Propoſitions are very uſeful in *Trigonometry*: I have made uſe of them in the eighth 'propoſition of my third book, to prove, that 'in a triangle the Sine total has the ſame proportion to the ſine of an angle, as the rectangle 'contain'd under the ſides, which form that 'angle, to double the triangle. I have us'd 'them likewiſe in the ſeventh, and divers other 'propoſitions.

PROPOSITION. XIV.

A PROBLEM.

To deſcribe a ſquare equal to a rectilineal figure given.



TO deſcribe a ſquare equal to the rectilineal A, make (*by the* 45. 1.) a rectangle BCDE equal to the rectilineal A. If the ſides CD, and CB were equal, we ſhould

ſhould have what we diſir'd: but being unequal, continue the line BC, ſo that CF may be equal to CD; and dividing the line BF, in the middle at the point G, deſcribe the ſemicircle FHB; this done, prolong DC to H. The ſquare of CH is equal to the rectilineal A. Draw the line GH.

Demonſtration.

The line BF is divided into two equal parts in G, and two unequal in C: therefore *(by the* 5.) the rectangle contain'd under BC, CF, or CD, that is to ſay, the rectangle BD, with the ſquare of CG, is equal to the ſquare of GB, or GH, which is equal to it. Now *(by the* 47. 1.) the ſquare of GH is equal to the ſquares of CG, and CH: therefore the rectangle BD, and the ſquare of CG, are equal to the ſquares of CG, and CH; and therefore taking away the ſquare of CG, which is common to both, there will remain the ſquare of CH equal to the rectangle BD, or, which is the ſame, the rectilineal A.

The USE.

'This Propoſition teaches us in the firſt place 'to reduce any rectilineal figures into ſquares; 'which being the chief meaſure of all ſuperfi- 'cies, becauſe its dimenſions are both equal, 'we can by this means take the magnitude of 'all ſorts of rectilineal figures. Again it helps 'us to find a middle proportional betwixt two

line

'lines given, as we ſhall ſee in the thirteenth
'Propoſition of the Sixth Book.

'*Ariſtotle* brings this Propoſition as an in-
'ſtance of a Formal Definition: for, *in his ſecond
'book, de Anima, ſect.* 12. diſtinguiſhing be-
'twixt a Formal and a Cauſal Definition, he ex-
'plains them thus. If, when tis demanded
'What it is to ſquare a Rectangle? anſwer be
'return'd, that it is to deſcribe a ſquare equal
'to a rectilineal; this anſwer contains the for-
'mal definition. But if it be ſaid, that it is to
'find a middle proportional betwixt two lines;
'this gives the Cauſal definition. For to find
'a middle proportional is the cauſe of making a
'ſquare equal to the rectilineal propos'd.

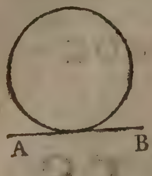
'This Propoſition may alſo be farther uſeful
'ſo the ſquaring of crooked figures; and alſo,
'as far as is poſſible, even the Circle it ſelf; for
'all ſorts of crooked figures may, at leaſt as far
'as is diſcernible by ſenſe, be reduc'd to recti-
'lineals. As for example, if we inſcribe in a
'circle a Polygone conſiſting of a thouſand
'ſides, there will be no ſenſible difference be-
'twixt it and the circle: therefore reducing this
'Polygone to a ſquare, we do, as far as our ſen-
'ſes are capable of judging, ſquare the circle.

THE

THE THIRD BOOK OF THE ELEMENTS OF EUCLID.

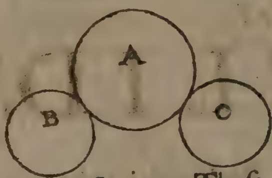
'THIS Third Book explains the properties of a Circle, and compares divers lines which may be drawn within, or without it's circumference. It considers likewiſe the circumſtances of circles, that cut each other, or touch a right line; and the differences of angles that are made either at the centers or circumferences. In fine it lays down the firſt principles for the eſtabliſhing the practical part of *Geometry*; for which the circle is moſt commodiouſly made uſe of in almoſt all Treatiſes of the *Mathematicks*.

DEFINITIONS.

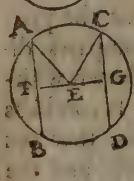


1. Thoſe Circles are equal, whoſe diameters or ſemidiameters are equal.

2. A line is ſaid to touch a circle, when, meeting with its circumference, it does not cut it. As the line AB.

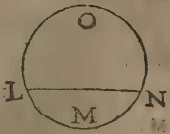


3. Circles touch, when meeting, they do not cut each other. As the circles A, B, and C.



4. Thoſe lines are equally remote from the center; when the perpendiculars, drawn from the center to the lines, are equal. ‘ As for ‘ example, if EF, and EG, perpen- ‘ diculars to the lines AB, and CD, ‘ be equal, AB and CD will be equally remote ‘ from the center; becauſe the diſtance ought ‘ always to be meaſur'd by perpendicular lines.

5. A



5. A ſegment of a circle is a figure terminated on one ſide by a right line, and on the other by the circumference of a circle As LON, LMN.

6. The angle of the ſegment is the angle which the circumference makes with the right line. " As the angles LNO, NLM.

7. An angle is in that ſegment in which are the lines that form it. ' As the angle FGH, is in the ſeg-' ment FGH.

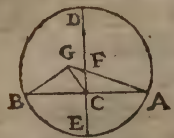
8. An angle is upon that arch, to which it is oppos'd, or which is as its baſe. " As the " angle FGH, is upon the arch FIH.

9. The Sector is a figure contain'd under two ſemidiameters, and the arch which ſerves them for a baſe. ' As the figure FIGH.

PROPOSITION I.

A PROBLEM.

To find the center of a Circle.



TO find the center of the circle AEBD, draw the line AB, and divide it in the middle at the point C; through which draw the perpendicular ED, which alſo divide into two equal parts at the point F, and that point F will be the center of the circle. If it be not, ſuppoſe the point G to be the center; and draw the lines GA, GB, and GC,

Demonſtration.

If the point G be the center, the triangles GAC, and GBC, will have the ſides GA, and GB equal,(*by the definition of a circle*:) and AC and CB will be equal, the line AB being divided in the middle at the point C, and CG being common, the angles GCB, and GCA will be equal (*by the* 8.1) and CG a perpendicular, not CD, which is contrary to the ſuppoſition. Therefore the center muſt of neceſſity be in the line CD. I add, that it muſt be at the point F, where it is divided into two equal parts: otherwiſe

wiſe the lines drawn from the center to the circumference would not be equal.

Coroll. The center of a circle is in that line, which falling perpendicularly upon another, divides it into two equal parts.

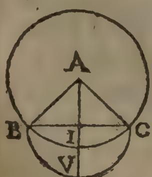
The USE.

'This Propoſition is neceſſary to demonſtrate 'thoſe that follow.

PROPOSITION II.

A Theorem.

A right line drawn from one point of the circumference to another, falls wholly within the circle



LET a line be drawn from the point B to the point C. I ſay, it will be wholly contain'd within the circle. To prove that it cannot fall without the circle, as BVC; having found the center of the circle A, draw the lines AB, AC, and AV.

Demonſtration.

The ſides AB, and AC, of the triangles ABC are equal: therefore *(by the* 5. 1.) the angle, ABC and ACB are equal, and ſince the angle AVC is an external angle in reſpect of the triangle

 angle

angle AVB, it is greater than the angle ABC, (*by the* 16. 1.) and then alſo it will be greater than the angle ACB. Therefore (*by the* 19.1.) in the triangle ACV, the ſide AC, oppos'd to the greater angle AVC, will be greater than AV: and by conſequence AV ought not to reach to the circumference of the circle, if the line BVC was a right line.

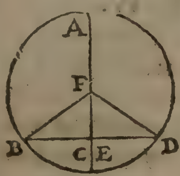
The USE.

'Tis by this Propoſition that they demon-'ſtrate, that a circle can touch a right line but 'in one place. For if the line touch'd two 'points of the circumference, it would be 'drawn from one of its points to another: and 'by conſequence, according to this Propoſition, 'would enter the circle; though by its defini-'tion, the line that touches ought not to cut 'the circumference. *Theodoſius* makes uſe of 'the ſame Demonſtration to prove, that a 'Globe can touch a plane only in one point; 'for otherwiſe the plane would enter within the 'Globe.

PROPOSITION III.

A THEOREM.

If the Diameter divide a line, which does not paſs through the center, into two equal parts, it will cut it at right angles; and if it cut it at right angles, it will divide it into two equal parts.



IF the diameter AC, cut the line BD, which does not paſs through the center F, into two equal parts at the point E, it will cut it at right angles. Draw the lines FB, and FD.

Demonſtration.

In the triangles FEB, and FED, the ſide EF is common; the ſides BE and ED are equal, becauſe the line BD is equally divided in E, and their baſes FB and FD are equal: therefore (*by the* 8. 1.) the angles BEF and DEF are equal, and by conſequence right angles. I add, that if the angles BEF and DEF be right angles, the line BD will be divided into two equal parts at E, that is to ſay, the lines BE and ED will be equal.

Demonſtration.

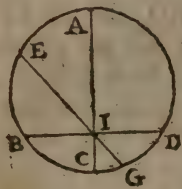
The triangles BEF and DEF are rectangular: therefore (*by the* 47. 1.) the ſquare of the

ſide DF will be equal to the ſquares of the ſides ED, and EF. Now the ſquares of BF and FD are equal, becauſe the lines are equal, therefore the ſquares of BE and EF are equal to the ſquares of DE and EF; and taking away the ſquare of EF, the ſquares of BE, and ED will be equal, and by conſquence the lines.

PROPOSITION IV.

A Theorem.

Two lines drawn within a circle cannot cut each other into two equal parts, unleſs they both paſs through the center.



If the lines AC and BD cut each other at the point I, which is not the center of the circle, they will not equally divide each other. Firſt, if one of thoſe lines, as AC, paſs through the center, 'tis evident it cannot equally be divided but at the center. But if neither paſs through the center, as BD and EG, draw the line AIC through the center. *Demonſtration.*

If the line AC divide the line BD into two equal parts in I, the angles AID and AIB will be

be right angles, (*by the* 3.) In like manner if the line EG was equally divided in I, the angle AIE would be a right angle; and consequently the angle AIB and AIE would be equal, which is impossible, one being part of the other. In a word the line AIC, which passes through the center, would be perpendicular to the lines BD and EG, if they were both equally divided at the point I.

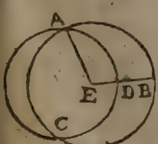
The USE.

These two Propositions are us'd in *Trigonometry*, to demonstrate, 'that the half of a chord 'of an arch is perpendicular to the semidiameter; and consequently, that it is the sine of 'half the arch. By these also they demonstrate 'that the sides of a triangle have the same proportion, as the sines of the opposite angles. We 'also make use of it to find the Eccentricity of 'the Circle, which the Sun describes in his an'nual motion.

PROPOSITION. V.

A THEOREM.

Circles that cut each other, have not the same center.



THE circles ABC, and ADC, which cut each other in A and C, have not the same center. If they had the same center,

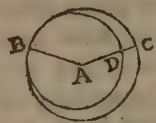
Sup-

ſuppoſe E, the lines EA and ED would be equal, (*by the definition of a circle;*) as alſo the lines EA, and EB: therefore the lines ED and EB would be equal, which is impoſſible, one being part of the other.

PROPOSITION VI.

A THEOREM.

Two circles that touch each other on the inner ſide have not the ſame center.



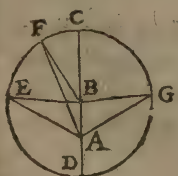
THE circles BD and BC, which touch each other on the inner ſide at the point B, have not the ſame center. For ſhould the point A be ſuppos'd to be the center of both the circles; the lines AB and AC, AB and AD, would be equal, (*by the definition of a circle,*) and conſequently the lines AD and AC would be equal, which is impoſſible, one being part of the other.

PROP.

PROPOSITION VII.

A Theorem.

If many lines be drawn from any one point within the circle, which is not its center, to the circumference: 1. that which passes through the center is the greatest: 2. the remainder of it, continu'd to the opposite part of the circumference, is the least: 3. that which is nearest to the greatest, exceeds those that are more remote: 4. There can be no more than two of them equal to each other.



Suppose many lines to be drawn from the point A, being not the center of the circle, to the circumference; and the line AC to pass through the center B: I will demonstrate, that it is greater than any of the other; for example, that it is greater than AF. Draw FB.

Demonstration.

The sides AB and BF of the triangle ABF, are greater than AF alone, (*by the 20. 1*) But BF and BC are equal, (*by the definition of a circle*:) therefore AB and BC, that is to say, AC, is greater than AF.

I add in the second place, that AD is the least;

least; for example, that it is less than AE, Draw BE. *Demonstration.*

The sides EA and AB are greater than BE alone, but BE is equal to BD, therefore EA and AB are greater than BD: taking therefore from both that which is common AB, AE will remain greater than AD.

Further, AF, which is nearer AC than AE, is also greater than it.

Demonstration.

The triangles FBA, and EBA, have the sides BF and BE equal, and BA is common to both: but the angle ABF is greater than the angle ABE: therefore (*by the* 24. 1.) AF is greater than AE,

Lastly, I say, that no more than two lines, that are equal to each other, can be drawn from the point A to the cicumference. Take the angles ABE and ABG equal; and draw the lines AE and AG.

Demonstration.

The triangles ABG, and ABE, having the sides BE and BG equal; the side AB common to both, and the angles ABE and ABG equal; therefore their bases AE, and AG will be equal, (*by the* 4. 1.) But all the lines that can be drawn either on one side or the other, will be either nearer AC, than AE, and AG; or more remote from it; and accordingly will be either greater or less than AG. Therefore there can no more

more than two lines equal betwixt themſelves be drawn from the point A to the circumference.

The USE.

‘*Theodoſius* advantagiouſly uſes this Propoſi-
‘tion to prove, that if from any point of the ſuperficies of a ſphere, which is not the pole of any certain circle, divers arches of greater circles be drawn to the circumference of that circle, that which paſſes through its pole will be the greateſt. For example: If from the pole of the world, which is diſtinct from the pole of the Horizon, (for the Zenith is its pole,) divers arches of greater circles be drawn to the circumference; the arch of the Meridian, which paſſes through the Zenith, will be the greateſt arch. This Propoſition is alſo brought to prove, that the Sun, when in his *Apogæum*, is moſt remote from the Earth.

PROP.

PROPOSITION VIII.

A Theorem.

If from a point taken without the circle, many lines be drawn to its circumference, 1. of all thoſe that extend to the concave circumference, that which paſſes through the center is the greateſt: 2. thoſe that lye neareſt to it, are greater than thoſe that are more remote: 3. among thoſe that fall upon the convex circumference, that which being continu'd paſſes through the center, is the leaſt: 4. the nearer to that are leſs than thoſe farther off: 5. there can be but two equal lines drawn from the ſame point either to the concave or convex circumference.

Suppoſe many lines were drawn from the point A to the circumference of the circle GCDE.

Firſt, the line AC, which paſſes through the center B, is the greateſt of all thoſe that reach to the concave circumference; for example, it is greater than AD. Draw the line BD.

Demonſtration.

In the triangle ABD, the ſides AB and BD are greater than AD alone; but the ſides AB and

and BC are equal to AB and BD: therefore AB and BC, or AC, is greater than AD.

2. AD is greater than AE.

Demonſtration.

The triangle ABD and ABE, have the ſide AB common to both, and the ſides BD and BE equal, and the angle ABD is greater than the angle ABE: therefore *(by the* 24. 1.*)* the baſe AD is greater than the baſe AE.

3. AF, which being continu'd paſſes through the center, is the leaſt of all thoſe that are drawn to the convex circumference LFIK; for example, it is leſs than AI. Draw IB.

Demonſtration.

In the triangle AIB the ſides AI and IB are greater than AB alone, *(by the* 20. 1.*)* therefore taking from both the equal lines BI and BF, AF will remain leſs than AI.

4. AI is leſs than AK. Draw the line BK.

Demonſtration.

In the triangles AIB and AKB, the ſides AK and KB are greater than the ſides AI, and IB, (*by the* 21. 1.) therefore taking from both the equal ſides BK, and BI, AI will remain leſs than AK.

5. There can be but two lines equal betwixt themſelves drawn. Take the angles ABL, and ABK; as alſo ABE, and ABG equal.

Demonſtration.

The triangles ABL, and ABK, will have their baſes

bases AL and AK equal, *(by the 4. 1.)* and by the same also AE and AG will be equal; but no other line can be drawn, that will not be either nearer to, or more remote from AF, or AC; and consequently, that will not be either greater or less than AK and AL, AE and AG.

PROPOSITION IX.

A Theorem.

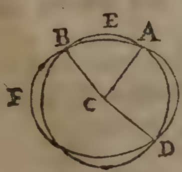
That point from whence three equal lines can be drawn to the circumference of a circle, is its center.

If the point were not the center of a circle, there could be but two equal lines drawn from it to the circumference, *(by the 7, and 8.)*

PROPOSITION X.

A Theorem.

Two circles cut each other only in two points.



If two circles AEBD, and ABFD, should cut each other in three points A, B, and D; find *(by the 1.)* the center C of the circle AEBD; and draw the lines CA, CB, and CD. *De-*

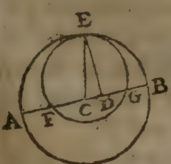
Demonstration.

The lines CA, CB, and CD, drawn from the center C to the circumference of the circle AEBD, are equal: but the same lines are also drawn to the circumference of the circle ABFD. therefore (*by the* 9.) the point C will be the center of the circle ABFD. So that two circles, which cut each other, will have the same center; which is contrary to the fifth Proposition

PROPOSITION. XI.

A THEOREM.

If two circles touch each other on the inside, a line drawn through both the centers, will also pass through the point where they touch.



IF the two circles EAB and EFG touch each other on the inside, at the point E; a line drawn through both their centers will pass through the point E. For if the point D was the center of the lesser circle, and C that of the greater, so that the line CD passing through both should not pass through the point E: draw the lines CE and DE.

Demonstration.

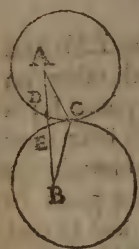
The lines DE, and DG, drawn from the

center of the leſſer circle D to its circumference, would be equal: and adding the line CD, the lines ED, and DC, would be equal to CG. Now ED and DC are greater than EC alone, (*by the* 20. 1.) and ſo CG will be greater than CE: yet C being the center of the greater circle, CE and CB are equal: therefore CG will be greater than CB, which is impoſſible.

PROPOSITION XII.

A Theorem.

If two circles touch each other on the outſide, a line drawn through both their centers, will paſs through the point where they touch.



IF the line AB, which does not paſs through the point C where the circles touch, be ſaid to be drawn from the center A to the center B; draw the lines AC and BC.

Demonſtration.

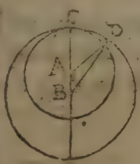
In the triangle ACB, the ſides AC and BC would not be greater than the ſide AB alone, (which is contrary to the 20. 1.) becauſe AD and AC, as alſo BE and BC, are equal.

PROP.

PROPOSITION XIII.

A THEOREM.

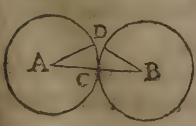
Two Circles can touch each other only in one point.



First, if two circles touch each other on the inside, they will touch but in one point only, the point C; which is markt out by the line BAC passing through both their centers, A, and B. For if they should touch likewise in the point D, draw the lines AD, BD.

Demonstration.

The lines AC and AD, drawn from the center of the lesser circle to its circumference, are equal: and adding AB, the lines BA, AC, and BA and AD, would be equal. Now BC and BD, drawn from the center of the greater circle to its circumference, will be equal: therefore the sides BA and AD will be equal to the side BD alone, which is contrary to the 20. 1.



Secondly, if two circles touch each other on the outside; drawing the line AB from one center to the other,

it will paſs through the point C, where the circles touch, (*by the* 12.) But if you ſay that they touch alſo at the point D: having drawn the lines AD and BD; the line BC and BD, AC and AD, being equal, the two ſides of a triangle taken together, would be equal to the third, which is contrary to the 20. 1.

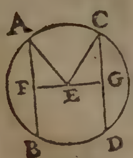
The USE.

'These four Propoſitions are very clear, and 'evident; and alſo neceſſary in *Aſtronomy*, 'when we make uſe of *Epicycles*, to explain the 'motions of the *Planets*.

PROPOSITION XIV.

A Theorem.

Equal lines drawn within a circle, are equally remote from the center; and thoſe that are equally remote from the center, are equal.



Suppoſing the lines AB and CD to be equal: I prove, that the perpendiculars EF and EG, drawn from the center, are alſo equal. Draw the lines EA and EC.

Demonſtration.

The perpendiculars EF and EG divide the lines AB and CD in the middles at the points

F

F and G, [*by the* 3.] therefore AE and CG are equal. The angles F and G are right angles: therefore [*by the* 47. 1.] the ſquare of EA is equal to the ſquares of EF and FA; as alſo the ſquare of EC is equal to the ſquares of EG and GC: but the ſquares of EA and EC are equal, becauſe the lines EA and EC are equal: therefore the ſquares of EF and FA are equal to the ſquares of EG and GC: and taking away the equal ſquares AF and CG, there will remain the ſquares of EF and EG equal; and conſequently the lines EF and EG, which are the diſtances of the lines AB and CD from the center, are equal.

But ſuppoſing the diſtances or perpendiculars EF and EG to be equal; I will prove after the ſame manner that the ſquares of EF and FA are equal to the ſquares of EG and GC; and taking away the equal ſquares of EF and EG, there will remain the ſquares of AF and CG equal. And therefore the lines AF and CG, and their double AB and CD, are equal.

PROPOSITION XV.

A THEOREM.

The Diameter is the greatest of all lines inscribed in a Circle; and of the rest that is the greatest which is nearest the Center.

THE diameter AB is the greatest of all lines that can be drawn in the circle GIDC. As for example, it is greater than CD; for draw the lines EC and ED.

Demonstration.

In the triangle CED, the sides EC and ED are greater than CD alone, [*by the* 20. 1.] but AE and EB, or AB, is equal to EC and ED; therefore the diameter AB is greater than CD.

Secondly, let the line GI be more remote from the center than the line CD; that is to say, let the perpendicular EH be greater than the perpendicular EF. I say that CD is greater than GI. Draw the lines EC, and EG.

Demonstration.

The squares of CF and FE [*by the* 47. 1.] are equal to the square of EC: but the square of EC is equal to the square of EG, and the square of

EG

EG equal to the ſquares of GH and HE: therefore the ſquares of CF and FE are equal to the ſquares of GH and HE; and taking from one ſide the ſquare of HE, and from the other the ſquare of EF, which is leſs than the ſquare of HE, the ſquare of CF will remain greater than the ſquare of GH. Therefore the line CF will be greater than the line GH; and the whole line CD, the double of CF, will be greater than GI, the double of GH.

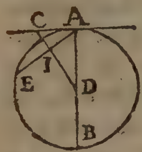
The USE.

'*Theodoſius* makes uſe of theſe two Propoſi-
'tions to demonſtrate, that in a ſphere the leſ-
'ſer circles are more remote from the center.
'I have alſo made uſe of them in *Aſtrolabes*. To
'theſe Propoſitions may likewiſe be referr'd that
'Mechanical propoſition of *Ariſtotle*, by which
'he ſhews, that the Rowers at the middle of a
'Gally have greater force, than thoſe that are
'at, either the fore, or hinder part thereof; be-
'cauſe the ſides of the Gally being crooked, the
'Oars of the middle part are longer, *i. e.* reach
'farther, than the reſt. The Demonſtrations
'relating to the *Iris*, or Rain-bow, do alſo ſup-
'poſe the truth of theſe propoſitions.

PROPOSITION XVI.

A Theorem.

A line drawn perpendicularly upon the extremity of the diameter, falls wholly on the out side of the circle, and touches it. But any other line drawn betwixt that and the circumference of the circle, enters within the circle, and cuts it.



LET the perpendicular AC be drawn upon the point A, which is the extremity of the diameter AB: I say first, that all the other parts of the same line, for example the point C, fall on the outside of the circle. Draw the line DC.

Demonstration.

Since the angle DAC of the triangle DAC is a right angle, DCA will be an acute: and (*by the* 19. 1.) the side DC will be greater than the side DA; therefore the line DC reaches beyond the circumference of the circle.

I add, that the line CA touches the circle, because that meeting with it at the point A it does not cut it, but all its points are on the outside of the circle.

I say also that no other line can be drawn from

from the point A below CA, which does not cut the circle. If there could, ſuppoſe EA to be ſuch an one; and from the point D draw a perpendicular to it, DI.

Demonſtration.

Since the angle DIA is a right angle, and the angle IAD an acute, AD will be greater than DI: therefore the line DI does not reach to the circumference, but the point I is within the circle.

The USE.

'Some Philoſophers uſe this Propoſition, but 'altogether in vain, to prove, that quantity is 'not diviſible *in infinitum*, or that there really 'are in the world ſuch things as *Zenonical*, i. e. '*abſolutely and in their own nature indiviſible* 'points. For the Propoſition does not, as they 'would have it, prove, that a circle touches a 'right line in a *Zenonical*, but in a *Mathema-* '*tical point*, which is nothing elſe but a quanti- 'ty conſidered without diſtinction of parts, that 'is to ſay, without conceiving them diſtinct and 'ſeparate one from the other, whether in reality 'it has ſuch parts or not making no matter. 'We can therefore take any quantity whatſoe- 'ver for a *Mathematical point*; which being 'once eſtabliſh'd, our circle will conſiſt of ſuch 'points, and will be *mathematically* perfect, 'provided it touch not a right line, but in a 'part equal to that quantity which we have ta-

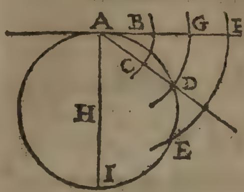
ken

'ken for a point. But if we afterwards take a 'leſs part for our *Mathematical point*, the circle 'which was exactly perfect according to the 'firſt ſuppoſition, will be imperfect in the ſe-'cond, and degenerate into a Polygone. I be-'lieve, tis as impoſſible to deſcribe a circle, 'that according to any ſuppoſition whatſoever 'ſhall be moſt exactly perfect, as it is to con-'ceive the leaſt poſſible quantity.

'Secondly, thoſe conſequences, which ſome 'men draw from this Propoſition relating to the 'angle of contact, which they take to be leſs 'than any rectilineal angle, are grounded upon 'this miſtake, that they imagine an angle to be 'a true quantity; the contrary of which may 'appear from hence, That the lines, that con-'tain an angle, being produc'd to any longi-'tude, the angle becomes not at all the greater. 'Further, it ought to be duly conſider'd, what 'we mean, when we ſay, that one angle is great-'er than another; for this is all we under-'ſtand, that a circle being deſcribed from the 'point of concourſe at any diſtance whatſoever 'the lines of that we call the greater angle will 'contain betwixt them a greater arch of that 'circle, than thoſe of that which we call the 'leſs; which is the ſole meaning of the Exceſs 'of one angle above another. From whence I 'infer, that the angle of contact can no more be 'compar'd with a rectilineal angle, than a ſu-

perficies

'superficies with a line, being at the same time 'both equal, and greater and less than a rectilineal angle. As for example; from the point 'A draw the line AD, 'making with AE a rectilineal angle; I say 'it is both greater and 'less than, and equal to, 'the angle of contact.



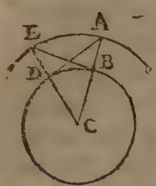
'For if we suppose divers circles describ'd 'from the point A, as the center, whereby to 'measure those angles; it is evident that, according to the arch drawn beyond the point D, 'that is the arch EF, the angle of contact 'is greater than the rectilineal angle. But on the 'contrary according to the arch CB, the rectilineal angle is the greater of the two. And 'lastly, according to the arch DG, passing 'through the point in which AD cuts the circumference, they are both equal. From 'whence it follows, that the angle of contact is 'at the same time both less and greater than, 'and equal to, the rectilineal angle: and consequently, they ought not at all to be compar'd together. In a word, Angles are no quantities; nor are they call'd less or greater one 'than another, but with respect to the arches 'which they contain: so that all the disputes 'about the angle of contact, and all the Paradoxes

'doxes, conclude nothing either for or against
'the divisibility of quantity; an Angle being
'no species, but only a property thereof.

PROPOSITION XVII.

A Problem.

From a point given to draw a line that may touch a Circle.



TO draw a line from the point A touching the circle BD, draw the line AC to its center; and at the point B draw a perpendicular BE, which may cut an arch of a circle, describ'd from the center C through the point A, at the point E. Draw also the lines EC, and AD. I say the line AD touches the circle in D.

Demonstration.

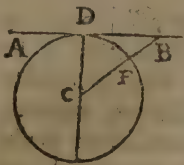
The triangles EBC and ADC have the same angle C; and the sides CD and CB, CE and CA equal, (*by the definit. of a Circle*:) and therefore they are equal in all respects, (*by the* 4. 1.) and the angles CBE and CDA are equal But the angle CBE is a right angle, therefore the angle CDA will be so too, and (*by the* 16.) the line AD will touch the circle.

PRO-

PROPOSITION XVIII.

A THEOREM.

A line drawn from the center of a circle to the point where a right line touches it, is perpendicular to that line.



IF the line CD be drawn from the center C to the point of contact D, CD will be perpendicular to AB. For if it be not, draw the line CB perpendicular to AB

Demonſtration.

Since the line CB is ſuppos'd to be perpendicular, the angle B will be a right angle, and conſequently CDB an acute, (*by the* 32. 1.) Therefore the line CB, oppos'd to the leſſer angle, will be leſs than CD, which is impoſſible; becauſe CF, which is but part of CB, is equal to CD.

PROP-

PROPOSITION XIX.

A Theorem.

If a line, perpendicular to the tangent, be drawn from the point of contact, it will pass through the center of the circle.

LET the line AB [*see Fig. preced.*] touch the circle at the point D, and the line DC be perpendicular to AB. I say, that DC passes through the center. For if it did not, drawing a line from the center to the point D, it would be perpendicular to AB, (*by the preceding*) and so there would be two perpendiculars drawn to the same point D of the same line, which cannot be.

The USE.

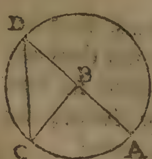
'The use of lines *Tangents* is very common 'in *Trigonometry*; upon which account it is that 'I have made a table, whereby to measure all sorts 'of triangles, as well spherical as rectilineal. In 'my *Opticks* likewise are divers propositions 'founded upon *Tangents*; as when is determin'd 'what part of a Globe is enlighten'd. The *pha-* '*ses* or Apparitions of the Moon are establish'd 'also upon the same doctrine; and that famous 'Problem of *Hipparchus*'s, by which he found 'the distance of the Sun, by the difference of the

'the true and apparent *Quadratures.* In Dial-
'ling the *Italian* and *Babylonian* hours are fre-
'quently defcrib'd by lines Tangents. Laftly
'we take the dimenfions of the Earth by a line
'that touches its fuperficies; and in the art of
'Navigation, take a Tangent line for our Hori-
'zon.

PROPOSITION. XX.

A THEOREM.

The angle at the center is double the angle at the circumference, which has the fame arch for its bafe.



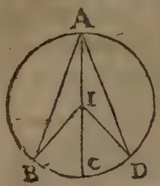
IF the angle ABC, which is at the center,, and the angle A DC, at the circumference, have the fame arch AC for their bafe, the firft will be double the fecond. This Propofition has three different cafes: the firft of which is, when the line ABD paffes through the center B, the line AB in one triangle concurring with the line BD of the other.

Demonftration.

The angle ABC is the external angle in refpect of the triangle BDC: therefore (*by the* 32. 1.) it is equal to both the angles D and C, which

which being equal, (*by the* 5. 1.) becauſe their ſides BC and BD are equal, the angle ABC is the double of either.

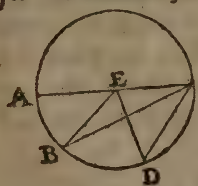
The ſecond caſe is, when one angle incloſes the other, but none of the lines that form them concur in one; as you ſee in the next figure. The angle BID is at the center, and the angle BAD at the circumference. Draw the line AIC through the center.



Demonſtration.

The angle BIC is double the angle BAC, and CID is double the angle CAD, (*by the preceding caſe* :) therefore the anlge BID is double the angle BAD,

The third caſe is, when it happens, that neither one angle incloſes the other, nor does any of the lines that form them, concur in one. *Which caſe is wholly omitted by my Author, but for the Readers ſatisfaction is here ſupplied*



Let the angle at the center be BED, and the angle at the circumference BCD, having the ſame arch for their baſe BD. I ſay, the angle BED is double the angle BCD. Draw the line EC, and continue it to the point A.

Demon-

Demonſtration.

The angle AED is double the angle ACD, (*by the* 1. *caſe* ;) and (*by the ſame*) the angle AEB is double the angle ACB : therefore the remainder of the one BED is double the remainder of the other BCD.

The USE.

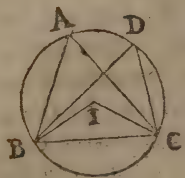
'That Problem, which is ordinarily pro-'pos'd, ſhewing how to deſcribe an *Horizontal* '*Dial* by one ſole opening of the Compaſs, is 'built in part on this Propoſition. And again, 'when we would determine the *Apogæum* of the 'Sun, or the excentricity of his Circle, by three 'obſervations, we ſuppoſe the angle at the cen-'ter to be double that at the circumference. '*Ptolomey* makes frequent uſe of this Propoſition 'to determine both the excentrick circle of the 'Sun, and the Epicycle of the Moon. The firſt 'Propoſition of the third book of *Trigonometry* 'is grounded alſo upon this here.

L PROP.

PROPOSITION XXI.

A THEOREM.

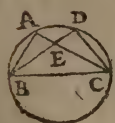
The angles, that are in the ſame ſegment of a circle, or that have the ſame arch for their baſe, are equal.



If the angles BAC and BDC are in the ſame ſegment of a circle, which is greater than a ſemicircle, they will be equal. Draw the lines BI and CI.

Demonſtration.

The angles A and D are each of them the half of the angle BIC, *(by the preceding,)* therefore they are equal. They have likewiſe the ſame arch BC for their baſe.



Secondly, let the angles A and D be in the ſame ſegment BAD, which is leſs than a ſemicircle; they will nevertheleſs be equal.

Demonſtration.

All the angles of the triangle ABE are equal to all the angles of the triangle DEC, *(by* 1. *Coroll. of the* 32. 1.) but the angles AEB and DEC are equal, *(by the* 15. 1.) Alſo the angles ECD and

and ABE are equal, (*by the preceding caſe*,) being in the ſame ſegment ABCD, greater than a ſemicircle; therefore the angles BAE and ED C are equal; which, the angles at E being equal, and conſequently (*by the Coroll. of the* 15. 1.) the lines AE and EC, making but one right line, as likewiſe DE and EB another, are the angles A and D, in the ſame ſegment ABCD, and having the arch BC for their baſe.

The USE.

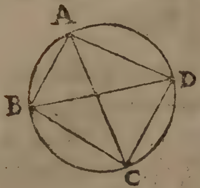
'This Propoſition is produc'd in *Opticks* to 'prove, that the line BC will appear of the 'ſame greatneſs, when tis view'd from A, and 'D, becauſe it is ſeen in both caſes under equal 'angles.

'The ſame Propoſition is us'd to deſcribe large 'circles without having their centers: for ex'ample, if we would make large Copper baſons of 'a ſpherical figure, ſuch as we might work upon 'in poliſhing Spectcales, and glaſſes to ſee at a 'great diſtance. For having made in Iron an an'gle BAC equal to that, which is contained in 'the ſegment ABC, and at the points B and C 'ſtrongly faſtn'd two ſmall iron pins; if the 'triangle BAC be mov'd ſo, that the ſide AB 'may always touch the pin B, and the ſide AC 'the pin C, the point A will deſcribe an arch 'of the circle ABCD. This manner of deſcribing a circle may alſo be us'd in making great Aſtrolabes.

PROPOSITION. XXII.

A Theorem.

Quadrilateral figures, inſcrib'd in a circle, have their oppoſite angles equal to two right angles.



LET a quadrilateral figure, or a figure of four ſides, be inſcrib'd in a circle, in ſuch ſort that all its angles may terminate at the circumference of the circle ABCD: I ſay the oppoſite angles BAD and BCD are equal to two right angles. Draw the diagonals AC, and BD.

Demonſtration.

All the angles of the triangle BAD are equal to two right angles. In ſtead then of the angle A BD put the angle ACD, which is equal to it(*by the* 21.) being in the ſame ſegment ABCD: and inſtead of the angle ADB, put the angle ACB, which is in the ſame ſegment of a cirele BCD A. Therefore the angles BAD, and the angles ACD and ACB, that is to ſay, the whole angle BCD, are equal to two right angles.

The USE.

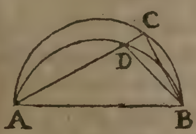
'*Ptolomey* makes uſe of this Propoſition to 'frame the table of Chords, or lines ſubtend-ing

'ing arches. I have alſo us'd the ſame in my 'third book of *Trigonometry*, to prove, that the 'ſides of an obtuſangle triangle would have the 'ſame proportion among themſelves as the ſines 'of the oppoſite angles.

PROPOSITION XXIII.

A THEOREM.

Two ſimilar ſegments of a circle, deſcrib'd upon the ſame line, are equal.



I call thoſe ſimilar ſegments of a circle, which contain equal angles; and I ſay, that if ſuch be deſcrib'd upon the ſame line AB, they will fall one upon the other, and not exceed each other in any part. For if either did exceed each other, as do the ſegments ADB, and ACB, they would not be ſimilar: to demonſtrate which, draw the lines ADC, BD, and BC.

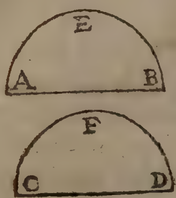
Demonſtration.

The angle ADB is an external angle in reſpect of the triangle DBC: therefore (*by the* 16, 1.) it is greater than the angle ACB, and by conſequence the ſegments ALB and ACB contain unequal angles, which I ſay is to be diſſimilar.

PROPOSITION XXIV.

A THEOREM.

Two similar segments of a circle describ'd upon equal lines, are equal.



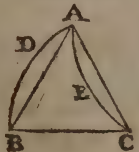
IF the segments of the circles AEB, and CFD, be similar, and the lines AB and BD equal, the segments also will be equal.

Demonstration.

Suppose the line CD to be plac'd upon the line AB, being suppos'd to be equal, they will not exceed each other; and then the segments AEB and CFD will be describ'd upon the same line, and therefore will be equal, (*by the preceding.*)

The USE.

'Crooked figures are frequently reduc'd to 'rectilineals by this Proposition. As for example: if two similar segments 'of a circle AEC, and ADB, be 'describ'd upon AB and AC, the 'equal sides of the triangle ABC: 'tis evident, that, transposing the 'segment AEC unto ADB, the 'triangle ABC is equal to the figure ADBCEA.

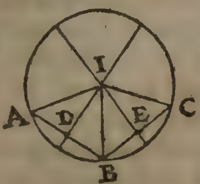


PROP.

PROPOSITION XXV.

A PROBLEM.

To compleat a circle, of which we have but a part.



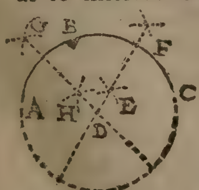
HAving the arch ABC given, to compleat the circle we muſt find its center; to which end draw the lines AB and BC, which having divided in the middle at the points E and D, draw their two perpendiculars EI and DI; which will meet at the point I, the center of the circle.

Demonſtration.

The center is in the line DI, (*by the coroll. of the* 1.) it is alſo in EI, (*by the ſame*;) therefore it muſt be at the point I.

The USE.

'This propoſition occurs very frequently: 'but ſometimes it is expreſs'd in other terms; 'as to inſcribe a triangle in a circle; or to de-



'ſcribe a circle through three 'points given, provided they 'be not plac'd in a right line 'Let the points propos'd be 'A, B, and C; and placing 'the foot of the compaſs at 'the point C, deſcribe two

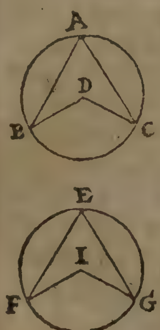
'arches F and E, at any diſtance whatſoever. 'Then remove the foot of the compaſs to the 'point B, and at the ſame diſtance deſcribe two 'other arches cutting the former in E and F; 'alſo from the point B, as the center, deſcribe 'at any diſtance the arches G and H, and at 'the ſame diſtance from the center A two other 'arches cutting them in G and H. Which done 'draw the lines through F and E, G and H, 'which ſhall cut each other at the point D, the 'center of the circle. The Demonſtration is 'obvious enough: for if you had drawn the 'lines AB, and BC, you had, by this opera-'tion, divided them equally and perpendicu-'larly. This Propoſition is exceeding neceſſa-'ry to deſcribe Aſtrolabes, and compleat cir-'cles, of which we have but three points. That 'Propoſition in *Aſtronomy*, which teaches how 'to find the *Apogæum*, and excentricity of the 'circle of the Sun, virtually contains this. And 'I alſo have made frequent uſe of it in my 'Treatiſe concerning the Cutting of Stones.

PROP.

PROPOSITION XXVI.

A THEOREM.

Equal angles, whether at the centers, or the circumferences of equal circles, have equal arches for their bases.



If the angles D and I, at the centers of equal circles ABC, and EFG, be equal; the arches BC and FG will be equal. For if the arch BC was greater or leſs than the arch FG, ſince the angles are meaſur'd by arches, the angle D would be greater or leſs than the angle I.

But if the equal angles be ſuppos'd to be at the circumferences of equal circles, as A and E; the angles which they encloſe at the centers, as D and I, being their doubles, will be likewiſe equal, and conſequently require equal arches for their baſes, BC and FG; which arches are likewiſe the meaſures of the angles A and E.

PROP.

PROPOSITION XXVII:

A Theorem.

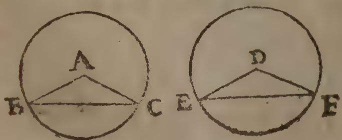
Angles, whether at the centers or circumferences of equal circles, having equal arches for their bases, are also equal.

IF the angles D and I (*fig. preced.*) at the centers of equal circles have equal arches BC and FG for their bases, they will be equal because their measures BC and FG are equal: And if the angles A and E, at the circumferences of equal circles have equal arches BC and FG for their bases, since the angles they enclose at the centers will be equal, they also that are the halves of those angles (*by the* 20.) will be equal.

PROPOSITION XXVIII.

A Theorem.

Equal lines, within equal circles, answer to equal arches.



IF the equal lines BC and EF be applied to equal circles, ABC, and

and DEF, they will be the chords of equal arches, BC, and EF. Draw the lines AB, AC, ED, EF. *Demonſtration.*

In the triangles ABC and DEF, the ſides AB and AC, DE and DF are equal, being the ſemidiameters of equal circles; and their baſes BC and EF are ſuppos'd equal, therefore (*by the* 8.1.) the angles A and D will be equal; and (*by the* 26.) the arches BC and EF will be alſo equal.

PROPOSITION XXIX.

A THEOREM.

The lines that ſubtend equal arches of equal circles are equal.

IF the lines BC and EF (*ſee fig. preced. Prop.*) ſubtend (or are the chords of) equal arches BC and EF in equal circles, they will be equal.

Demonſtration.

The arches BC and EF are equal, and parts of equal circles; therefore (*by the* 27) the angles A and D will be equal. Therefore in the triangles ABC, DEF, the ſides AB, AC, DE, and DF being equal, as alſo the angles A and D; the baſes BC, EF will be equal, (*by the* 4. 1.)

The USE.

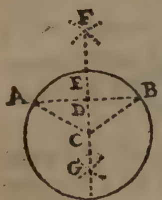
Theodoſius by the 28 and 29 demonſtrates, that

'that the arches of the circles of the *Italian* and
'*Babylonian* hours, contain'd between two pa-
'rallels, are equal. We have alſo demonſtrat-
'ed after the ſame manner, that the arches of
'the circles of the Aſtronomical hours, contain'd
'between two lines parallel to the *Equator*, are
'likewiſe equal. Theſe Propoſitions are almoſt
'of continual uſe in ſpherical *Trigonometry*, and
'alſo in *Dialling*.

PROPOSITION XXX.

A PROBLEM.

To divide an arch of a circle into two equal parts.



SUppoſe the arch AEB was to be divided into two equal parts, Place the foot of the compaſs at the point A, and deſcribe two arches F and G; then removing it to the point B at the ſame diſtance deſcribe other two arches, cutting the former in F and G; the line GF will cut the arch AB equally at the point E. Draw the line AB.

Demonſtration.

By this operation you have divided the line AB into two equal parts. For ſuppoſe there were drawn

drawn the lines AF, BF; AG, and EG; (which I have not done, leaſt the figure ſhould appear confus'd,) the triangles FGA and FGB would have all their ſides equal, therefore (*by the* 8. 1.) the angles AFD, & BFD would be equal. Again the triangles DFA and DFB have the ſide DF, common, the ſides AF and BF equal, and the angles DFA and DFB equal: therefore (*by the* 4. 1.) the baſes AD and BD are equal, and alſo the angles ADF and BDF. We have therefore divided the line AB equally and perpendicularly at the point D. Therefore (*by the* 1.) the center of the circle is in the line FG: Suppoſe it then to be the point C, and draw the lines CA and CB; all the ſides of the triangles ACD and BCD are equal: therefore [*by the* 8, 1.] the angles ACD and BCD are equal, and [*by the* 26.] the arches AE and EB.

The USE.

'Having frequent occaſion to divide an arch 'into two equal parts, the exerciſe of this Pro-'ſition is very common. 'Tis thus that we 'divide the Mariners compaſs into 32 winds: 'for having drawn two diameters cutting each 'other at right angles, we divide the circle in-'to four, and ſubdividing each quarter in the 'middle, we have eight parts; and again ſub-'dividing thoſe twice, we make 32. We have

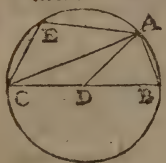
alſo

'also occasion for the same operation in the di-
'viding a semicircle into 180 degrees; and be-
'cause to compleat that division we are oblig'd
'to divide an arch into three equal parts, all *Geo-
'metricians* have sought after a method of do-
'ing that *Geometrically*, but have not yet been
'so happy as to find one.

PROPOSITION XXXI.

A Theorem.

The angle in a semicircle is a right angle, that which is in a segment greater than a semicircle is an acute, and that which is in a lesser segment is an obtuse.



If the angle BAC be in a semicircle, I will prove that it is a right angle. Draw the line DA

Demonstration.

The angle ADB being an external angle in regard of the triangle DAC, is equal to both the internals DAC, and DCA (*by the* 32.1.) and those being equal (*by the* 5.1.) because the sides DA and DC are equal it will be double the angle DAC. In like manner the angle ADC is double the angle DAB: therefore the two angles ADB, and ADC, which are equal to two right angles, are double the

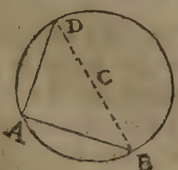
the whole angle BAC, and conſequently the angle BAC is a right angle.

Secondly, the angle AEC, which is in the ſegment AEC leſs than a ſemicircle, is an obtuſe angle. For in the quadrilateral figure AB CE, the two oppoſite angles E and B are equal to two right angles, *(by the 22.)* but the angle B is an acute; therefore the angle E will be an obtuſe.

Thirdly, the angle B, which is in the ſegment ABC greater than a ſemicircle, is an acute; becauſe in the triangle ABC, the angle BAC is a right angle.

The USE.

'*Mechanicks* make uſe of this Propoſiton 'to try if their Squares be juſt; 'for having deſcrib'd a ſemicircle BAD, they lay down 'the point A of their ſquare B 'AD upon the circumference, 'and one of its ſides AB upon 'the point of the diameter B: 'and then the other branch AD ought to paſs 'preciſely to the point D, which is the other 'extreme of the diameter.



'*Ptolomey* uſes this Propoſition to compoſe his 'table of Chords or Subtendants, of which he 'has occaſion in his *Trigonometry*.

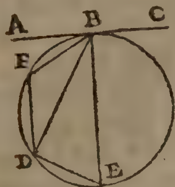
'There is alſo a method of raiſing a perpendicular

'dicular at the end of a line, grounded upon
'this Propoſition: For example to raiſe a perpen-
'dicular at the point A of the line AB. I place
'the foot of the compaſs upon the point C ta-
'ken any where, and deſcribe a circle through
'the point A, cutting the line AB at the point
'B. Then I draw the line BCD; and ſo 'tis
'evident, that the line AD is in a ſemicircle.

PROPOSITION XXXII.

A THEOREM.

A line cutting a circle at the point of contact, makes, with the tangent, angles, equal to thoſe in the alternate ſegments.



LET the line BD cut the circle at the point B, which is that where the line AB touches it. I ſay the angle CBD, made by the line BD and the tangent ABC, is equal to the angle F in the alternate ſegment BFD; and that the angle ABD is equal to the angle E in the ſegment BED.

Firſt, if the line paſs through the center, as the line BE, it will make with the tangent two right angles, (*by the* 18.) and the angles of the ſemicircles would be alſo right angles, (*by the* pre-

preceding,) therefore in this caſe the propoſition would be true. But if the line do not paſs through the center, as BD; draw the line BE through the center, and joyn the line DE.

Demonſtration.

The line BE makes, with the tangent, two right angles; and all the angles of the triangle BDE are equal to two right angles, (*by the* 32. 1.) therefore taking away the right angles CBE, and D which is in the ſemicircle, and likewiſe the angle EBD which is common to both, there will remain the angle ABD equal to the angle E.

Again, the angle CBD is equal to the angle F; becauſe in the quadrilateral figure BFDE, which is inſcrib'd in a circle, the oppoſite angles E and F are equal to two right angles, [*by the* 22.] but the angles ABD and CBD are alſo equal to two right angles, [*by the* 13: 1.] and the angles ABD and E are equal, as I have now demonſtrated: therefore the angles CBD and F are equal.

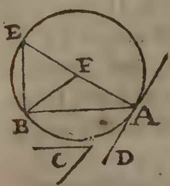
The USE.

'This Propoſition is neceſſary to prove that 'which follows,

PROPOSITION XXXIII.

A PROBLEM.

Upon a line given to deſcribe a ſegment of a circle capable of an angle given.



LET it be propos'd to deſcribe a ſegment of a circle upon the line AB capable of the angle C. Make the angle BAD equal to the angle C, and draw AE perpendicular to AD; make alſo the angle ABF equal to the angle BAE: and in fine, from the point F, where BF and AE concur; at the diſtance BF or FA, deſcribe a circle. The ſegment BEA is capable of an angle equal to the angle C.

Demonſtration.

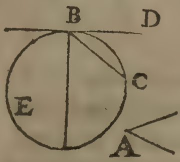
The angles BAF and ABF being equal, the lines FA and FB are equal, [*by the* 6. 1.] and the circle, which is deſcrib'd from the center F, by A, paſſes by B: Now the angle DAE being a right angle, the line DA touches the circle in A, [*by the* 16.] therefore the angle contain'd in the ſegment BEA, as the angle E, is equal to the angle DAB, that is the angle C, [*by the preceding.*] But if the angle given be an obtuſe, we muſt take an acute, its complement to 180 degrees.

PROP.

PROPOSITION XXXIV.

A PROBLEM.

A circle being given, to cut a ſegment in it capable of a certain angle.



TO cut a ſegment of the circle BCE capable of the angle A, draw [*by the* 17.] the tangent BD, and make the angle DBC equal to the angle A. Tis evident [*by the* 32.] that the ſegment BEC is capable of an angle equal to DBC, and conſequently to the angle A.

The USE.

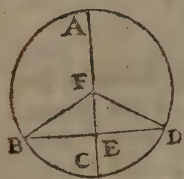
'I have made uſe of this Propoſition to find 'Geometrically the excentricity of the Annual 'circle of the Sun, and his *Apogæum*, having 'three obſervations given. Tis uſed likewiſe 'in *Opticks*, to find a point where two unequal 'lines propos'd may appear equal, or under 'equal angles, by making upon each line ſeg-'ments which will contain equal angles.

PROPOSITION XXXV.

A THEOREM.

If two lines cut each other within a circle, the rectangle contain'd under the parts of one is equal to the rectangle contain'd under the parts of the other.

First, if the two lines cut each other in the center, they will be both equal, and both equally divided; ſo that in that caſe it is evident, the rectangle contain'd under the parts of one, will be equal to the rectangle contain'd under the parts of the other.



Secondly, if one of the lines paſs through the center F, as AC, and divide the line BD, into two equal parts at the point E: I ſay the rectangle contain'd under AE and EC is equal to the rectangle contain'd under BE and ED, that is to ſay, to the ſquare of BE. The line AC is perpendicular to BD, [*by the* 3.]

Demonſtration.

Since the line AC is divided equally in F, and unequally in E, the rectangle contain'd under AE, and EC, with the ſquare of FE, is equal to

to the ſquare of FC or FB [*by the* 5. 2.] Now the angle E being a right angle, the ſquare of FB is equal to the ſquares of BE and FE; therefore the rectangle under AE, EC, with the ſquare of FE, is equal to the ſquares of BE and EF: and taking away the ſquare of EF, there remains the rectangle under AE, EC, equal to the ſquare of BE.

Thirdly, let the line paſs through the center F, and divide the line CD into unequal parts at the point E: draw FG perpendicular to CD, and [*by the* 3.] the lines CG and GD will be equal.

Demonſtration.

Since the line AB is divided equally in F and unequally in E, the rectangle contain'd under AE, EB, with the ſquare of EF, is equal to the ſquare of FB, or FC, [*by the* 5. 2.] Inſtead of the ſquare of EF put the ſquares of FG and GE, which are equal to it, [*by the* 47. 1.]

In like manner the line CD being divided equally in G, and unequally in E; the rectangle under CE, ED, with the ſquare of GE, is equal to the ſquare of GC. Add the ſquare of GF; the rectangle under CE, ED, with the ſquares of GE and GF, will be equal to the ſquares of CG and GF, that is to ſay, [*by the* 47. 1.] to the ſquare of FC. Therefore the rectangle under

der AE, EB, with the ſquares of EG and GF, is equal to the rectangle under CE, ED, with the ſame ſquares: and conſequently taking away the ſame ſquares from both, the rectangle AE, EB, will be equal to the rectangle CE, ED.

Fourthly, if the lines CD and HI, cut each other in E, neither of the two paſſing through the center: I ſay, the rectangle CE, ED is equal to the rectangle HE, EI. For drawing the line AFB, it is plain the rectangles CE, ED, and HE, EI, are both equal to the rectangle AE, EB, [*by the preceding caſe*;] therefore they are equal betwixt themſelves.

The USE.

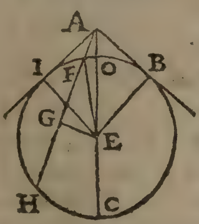
We are taught by this Propoſition a method ‘ of finding a fourth proportional to three lines ‘ given, or a third proportional to two.

PROP.

PROPOSITION XXXVI.

A THEOREM.

If from a point taken without the circle a line be drawn to touch, and another to cut the circle; the ſquare of the Tangent will be equal to the rectangle contain'd under the whole ſecant, and the external line.



SUppoſe the line AB to be drawn from the point A, taken without the circle, to touch the circle in B; and the line AC, or AH cutting it. The ſquare of AB will be equal to the rectangle contain'd under AC, and AO as alſo the rectangle contain'd under AH, and AF. If the ſecant paſs through the center, as AC, draw the line EB.

Demonſtration.

Since the line OC is divided in the middle at the point E, and the line AO added to it; the rectangle contain'd under AO and AC, with the ſquare of OE or EB, will be equal to the ſquare of AE, [*by the* 6.2.] Now the line AB is ſuppos'd to touch the circle at the point B; therefore [*by the* 18.] the angle B is a right an-

 gle,

gle, and (*by the* 47. 1.) the ſquare of AE is equal to the ſquares of EB and AB; therefore the rectangle under AC and AO, with the ſquare of EB, is equal to the ſquares of EB and AB: and taking away the ſquare of EB from both, the rectangle under AC, AO will be equal to the ſquare of AB.

Secondly, ſuppoſe the ſecant AH not to paſs through the center; and draw the line EG perpendicular to FH, which will divide in the middle the line FH at the point G; draw alſo the line EF.

Demonſtration.

The line FH being divided equally at the point G, and the line AF being added to it; the rectangle contain'd under AH, AF, with the ſquare of FG, will be equal to the ſquare of AG. Add to both the ſquare of EG: the rectangle under AH, AF, with the ſquares of FG and GE, that is (*by the* 47. 1.) the ſquare of FE, or EB, will be equal to the ſquares of AG and GE, that is, (*by the* 47. 1.) the ſquare of AE. Further, the ſquare of AE (*by the ſame*) is equal to the ſquares of EB and AB: therefore the rectangle contain'd under AH, AF, with the ſquare of BE, is equal to the ſquares BE and AB: and taking away the ſquare of BE from both, the rectangle contain'd under AH, AF will be equal to the ſquare of AB.

Coroll. 1. If you draw divers ſecants from the
ſame

ſame point, as AC and AH, the rectangles under AC and AO, AH and AF, will be equal betwixt themſelves, ſince they are both equal to the ſquare of AB.

Coroll. 2. If you draw two tangents from the ſame point, as AB, AI, they will be equal: becauſe the ſquares will be equal to the ſame rectangle under AC, and AO, and conſequently betwixt themſelves; as alſo the lines.

PROPOSITION XXXVII.

A THEOREM.

If the rectangle contain'd under the ſecant and the external line be equal to the ſquare of a line that falls upon the circle, that line will touch the circle.

SUppoſe the ſecant to be AC or AH, and the rectangle AC,AO; or AH,AF, (*ſee fig. preced.*) to be equal to the ſquare of the line AB; the line AB will touch the circle. Draw the tangent AI, (*by the* 17.) and the line IE.

Demonſtration.

Since the line AI touches the circle, the rectangle AC, AO; or AH, AF, will be equal to the ſquare of AI. But the ſquare of AB is ſuppos'd to be equal to either of thoſe rectangles; therefore the ſquares of AI and AB are equal, and conſequently the lines AI and AB. Therefore the

the triangles ABE and AIE, having all ſides equal, will be equiangular, (*by the* 8. 1.) and becauſe the angle AIE is a right angle (*by the* 18.) the line AI being a tangent, the angle ABE will be a right angle, and the line AB a tangent, (*by the* 16.)

The U S E.

'*Maurylocus* makes uſe of this Propoſition 'to find the diameter of the Earth. For obſerv-'ing from the top of a mountain OA, the ſu-'perficies of the Earth by the line BA, he takes 'notice of the angle OAB, made by the line A 'B, and a perpendicular AC: and by *Trigonome-'try* calculates the length of the line AB. Then 'multiplying AB by AB to have its ſquare, 'he divides the product by AO the height of 'the mountain, which gives the quotient AC, 'the diameter of the earth, with the height of 'the mountain; from which having ſubducted 'AO, there will remain OC the diameter of 'the earth. This Propoſition ſerves alſo to prove 'the fifth of the third book of *Trigonometry*.

THE

THE FOURTH BOOK OF THE ELEMENTS OF EUCLID.

'THis fourth Book is exceeding uſeful in 'Trigonometry. For by inſcribing Poly-'gons in a Circle, we learn the methods of 'compoſing the Table of Subtendants, Tan-'gents, and Secants; a practice moſt neceſſary 'for taking all ſorts of Dimenſions.

'Again, by inſcribing Polygons in a circle, 'we find the divers Aſpects of the Stars, which 'alſo take their names from thoſe Polygons.

'Thirdly, the ſame Operations give us the '*Quadrature* of the *Circle*, as exact as is needful. 'And by them we alſo demonſtrate, that Cir-'cles are in the duplicate proportion of that of 'their Diameters.

'Fourthly, Military Architecture does fre-quently

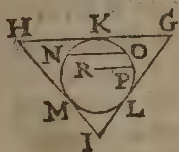
'quently require to inſcribe Polygons in a Cir-
'cle, to compoſe the deſigns and platforms of
'regular Fortifications.

DEFINITIONS.

1. A Rectilineal figure is inſcrib'd in a circle, or a circle is deſcrib'd about it, when all its angles are in the circumference of the ſame circle.

'As the triangle ABC is inſcrib'd in a circle, and the circle is deſcrib'd 'about the triangle; becauſe its angles A, B, 'and C, do all terminate at the circumference. 'The triangle DEF is not inſcrib'd in the cir-'cle, becauſe the angle D does not terminate at 'the circumference of the circle.

2. A rectilineal figure is deſcrib'd about a circle, and the circle inſcrib'd within that figure, when all the ſides of the figure touch the circumference of the circle. "As the trian-'gle GHI is deſcrib'd about 'the circle KLM becauſe its 'ſides touch the circumference 'of the circle in K, L, and M.



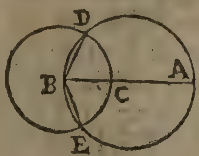
3. A line is apply'd to, or inſcrib'd in a circle,

cle, when its two extreams touch the circumference of the circle. " As the line NO. But " the line RP is not inſcrib'd in the circle.

PROPOSITION I.

A PROBLEM.

To inſcribe in a circle a line, that does not exceed its Diameter.



LET a line be propos'd to be inſcrib'd in the circle AEBD, not exceeding its diameter. Take the length of the Line propos'd upon the diameter; for example, let it be BC. Place the foot of the compaſs upon the point B, and deſcribe a circle at the diſtance of BC, which may cut the circle AEBD in D and E. Then draw the line BD or BE. 'Tis evident they are equal to BC, (*by the definition of a circle.*)

The USE.

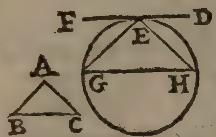
" This Propoſition is neceſſary for the performance of what is requir'd in the following.

PROP.

PROPOSITION II.

A PROBLEM.

To inſcribe in a circle a triangle equiangular to another triangle.



LET the circle be EGH, in which a triangle is to be inſcrib'd, equiangular to the triangle ABC. Draw the tangent FED, *(by the* 17. 3.*)* and at the point of contact E make the angle DEH equal to the angle B, and the angle FEG equal to the angle C, [*by the* 23. 1.] and draw the line GH; the triangle EGH will be equiangular to the triangle ABC.

Demonſtration.

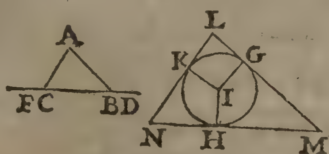
The angle DEH is equal to the angle EGH of the alternate ſegment, [*by the* 32. 3.] But the angle DEH is equal to the angle B, and conſequently the angles B and G are equal. By the ſame reaſon the angles C and H are alſo equal, and [*by Coroll.* 2. *of the* 32. 1.] the angles A and GEH will be equal. Therefore the triangles EGH and ABC are equiangular.

PROP.

PROPOSITION III.

A PROBLEM.

To describe a triangle about a circle equiangular to another triangle.



IF you would describe a triangle equiangular to the triangle ABC about the circle GKH Continue one of the sides of the triangle given BC to D and F, and make the angle GIH equal to the angle ABD, and HIK equal to the angle ACF: then draw the tangents LGM, LKN, and NHM, through the points G, K, and H. These tangents will concur; because the angles IKL and IGL being right angles, if you should draw a line KG, the angles KGL and GKL would be less than two right angles: therefore the lines GL and KL must concur, [*by the* 11. *Axiom*]

Demonstration.

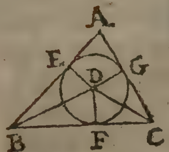
All the angles of the quadrilateral GIHM are equal to four right angles, because it may be divided into two triangles. The angles IGM and IHM, which are made by the tangents, are

are right angles; therefore the angles M and I are equal to two right angles, as are alſo the angles ABC and ABD. But the angle GIH is equal to the angle ABD, therefore the angle M will be equal to the angle ABC. By the ſame reaſon the angles N and ACB are equal, and therefore the triangles LMN and ABC are equiangular.

PROPOSITION IV.

A PROBLEM.

To inſcribe a Circle in a Triangle.



IF you would inſcribe a circle in the triangle ABC, divide the angles ABC and ACB into two equal parts, [*by the* 9. 1.] drawing the lines BD and CD, which will concur at the point D. This done, from the point D draw the perpendiculars DE, DF, and DG, which will be equal; ſo that a circle deſcrib'd from the center D, at the diſtance DE, will paſs through F and G.

Demonſtration.

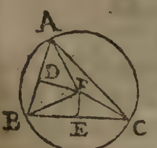
The triangles DEB and DFB have the angles DEB and DFB equal, being both right angles: the angles DBE and DBF are alſo equal, the angle

angle ABC having been divided into two equal parts; and the ſide DB is common: therefore (*by the 26. 1.*) the triangles will be equal in all reſpects, and the ſides DE and DF will be equal. After the ſame manner might I demonſtrate the ſides DF and DG to be equal. 'Tis poſſible therefore to deſcribe a circle, which ſhall paſs through the points E, F, and G; and becauſe the angles E, F, and G are right angles, the ſides AB, AC, and BC will touch the circle, which by conſequence is inſcrib'd in the triangle.

PROPOSITION V.

A PROBLEM.

To deſcribe a Circle about a Triangle.



IF you would deſcribe a circle about the triangle ABC, divide the ſides AB and BC into two equal parts, at the points D and E, drawing the perpendiculars DF and EF, which will concur at the point F. Which done, if you deſcribe a circle from the center F, at the diſtance FB, it will paſs through A and C; that is to ſay, the lines FA, FB, and FC, are equal.

Demonſtration.

The triangles ADF and BDF have the ſide

DF common, and the ſides AD and DB equal, the ſide AB having been divided equally in D; and the angles at D are equal, being right angles. Therefore (*by the* 4. 1.) the baſes *AF* and *BF* are equal; as alſo the baſes *BF* and *CF*.

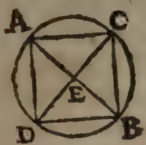
The USE.

'I have frequent occaſion to inſcribe a triangle in a circle; as, for inſtance, in the firſt 'propoſition of my 3. book of Trigonometry. 'This performance alſo is neceſſary for the meaſuring the *area* of a triangle; and upon many 'other occaſions.

PROPOSITION VI.

A PROBLEM.

To inſcribe a ſquare in a circle.



TO inſcribe a ſquare in the circle ACBD, draw the diameter AB, and perpendicular to it the line DC paſſing through the center E; then draw the lines AC, CB, BD, DA, and you will have inſcrib'd in the circle the ſquare ACBD.

Demonſtration.

The triangles AEC and CEB have their ſides equal, and the angles AEC and CEB equal, being

being both right angles: therefore their bases AC and CB are equal, (*by the* 4. 1.) Further, becauſe the ſides AE and EC are equal, the angles EAC and ECA will be equal: and the angle E being a right angle, they will be half-right angles, (*by the* 32. 1.) therefore the angles ECB is half a right angle, and conſequently the angle ACB will be a right angle. And the ſame reaſon holds for all the reſt: therefore the figure ACBD is a ſquare.

PROPOSITION VII.

A PROBLEM.

To deſcribe a ſquare about a circle.

HAving drawn the two diameters AB, and CD, which cut each other perpendicularly at the center E, draw the tangents *F*G, GH, HI, and I*F*, by the points A, D, B, C, and you will have deſcrib'd the ſquare *F*GHI, about the circle ADBD.

Demonſtration.

The angles E and A are right angles, therefore (*by the* 27. 1.) the line, *F*G and CD are parallels. After the ſame manner I may prove that CD and HI, *FI* and AB, AB and DH, are parallels. Therefore the figure *F*CDG is a

parallelogram, and *(by the* 34. 1.) the lines FG and CD are equal, as alſo the lines CD and IH, FI and AB, AB and GH; and conſequently the ſides of the figure FG and HI are equal. Further, ſince the lines FG and CD are Parallels, and the angle CDG is a right angle, the angle G will alſo be a right angle, [*by the* 29. 1.) After the ſame manner I may demonſtrate the angles F, H, and I, to be right angles. Therefore the figure FGHI is a ſquare, whoſe ſides touch the circle.

PROPOSITION VIII.

A Problem.

To inſcribe a Circle in a Square.

IF you would inſcribe a Circle in the Square FGHI, [*ſee the fig. preced.*] divide the ſides FG, GH, HI, IF, in the middles at the points A, D, B, C, and draw the lines AB and CD, which may cut each other at the point E. I demonſtrate that the lines EA, ED, EB, and EC are equal, and the angles A, D, B, C, right angles: and that therefore you may deſcribe a circle from the center E, which will paſs through A, D, B, and C, and touch the ſides of the ſquare FGHI,

Demon-

Demonſtration.

Since the lines AB and GH do conjoyn the lines AG and BH, which are parallel and equal, they alſo will be parallel and equal: therefore the figure AGDE is a parallelogram; and the lines AE and GD, AG and ED are equal: and AG and GD being equal, AE and ED will be equal alſo. 'Tis after the ſame manner that the lines AE, EC, EB, are prov'd equal. Further, AG and CD being parallel, and the angle G a right angle, the angle D will be ſo likewiſe. Therefore the circle ADBC may be deſcrib'd from the center E, which will paſs through the points A, D, B, C, and touch the ſides of the ſquare.

PROPOSITION IX.

A Problem.

To deſcribe a Circle about a Square.

To deſcribe a circle about the ſquare ACBD, [*ſee fig. in Prop.* 6.] draw the diagonals AB, and CD, which will cut each other at the point E; the point E will be the center of the circle, which will paſs through the points A, C, B, D. It ought therefore to be demonſtrated, that the lines AE, EB, CE, and ED are equal.

 Demon-

Demonstration.

The sides AC and CB are equal, and the angle C is a right angle; therefore the angles BAC and ABC are equal, (*by the* 5. 1.) and half right angles, (*by the* 32. 1.)

After the same manner I demonstrate, that the angles ACD, ADC, BDC, and BCD, are half right angles. Therefore the triangle AEC having the angles EAC and ECA half right angles, and consequently equal, will have also (*by the* 6. 1.) its sides AE and EC equal. The same may be prov'd of the lines EC and EB, EB and ED, that they likewise are equal.

The USE.

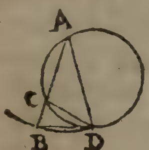
'We shew in the 12 Book, that Polygons 'inscrib'd in a circle, degenerate into circles; 'and as these Polygons are always in the duplicate proportion of that of their diameters, so 'likewise are circles. In practical *Geometry* 'we have frequent occasion to inscribe a square 'and other Polygons in a circle, or to describe 'them about it, to reduce a circle to a square.

PROP.

PROPOSITION. X.

A PROBLEM.

To describe an Isosceles *(or equicrural triangle) having its angles at the base, each of them double to the third angle.*



TO describe an *Isosceles* ABD, having each of its angles ABD and ADB, double the angle A, divide the line AB (*by the* 11. 2.) so that the square of AC may be equal to the rectangle under AB and BC; and from the center A at the distance AB describe the circle BD, in which inscribe BD equal to AC; and drawing the line DC describe a circle about the triangle ACD, (*by the* 5.)

Demonstration.

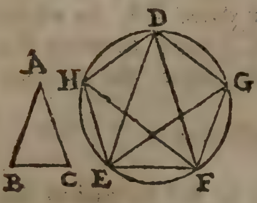
Since the square of AC or BD is equal to the rectangle contain'd under AB and BC, the line BD will touch the circle ACD at the point D, (*by the* 37. 3.) therefore the angle BDC will be equal to the angle A, being in the alternate segment CAD, (*by the* 32. 3.) Now the angle BCD, being an external angle in respect of the triangle ACD, is equal to the angles A and

CDA; therefore the angle BCD is equal to the angle BDA. Further, the angle ADB is equal to the angle ABD, (*by the* 5. 1.) therefore DCB and DBC are equal, and (*by the* 6. 1.) the sides BD and DC will be equal: and since BD is equal to AC, the sides AC and CD will be equal; and so likewise the angles A and CDA. Therefore the angle ADB is double the angle A.

PROPOSITION XI.

A Problem.

To inscribe a regular Pentagon in a circle.



TO inscribe a Regular Pentagon in a circle, describe (*by the* 10.) an *Isosceles* ABC, having each of its angles ABC, ACB, at the base, double the angle A. Inscribe in the circle the triangle DEF equiangular to ABC: then divide the angles DEF and DFE into two equal parts, drawing the lines EG and FH. Lastly, joyn the lines DH, DG, GF, EH, and you will have describ'd a regular Pentagon; that is to say, a Pentagon having equal sides, and equal angles.

Demon-

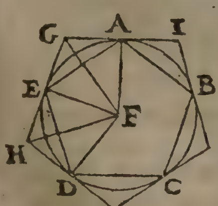
Demonſtration.

The angles DEG, GEF, DFH, and HFE, being the halves of the angles DEF and DFE, each of which is double to the angle EDF, are equal to the angle EDF: and conſequently the five arches, which are their baſes, are equal, *(by the* 26. 3.) and the lines DH, HE, EF, FG, and GD, are equal, (*by the* 29. 3.) Secondly, the angles DGF, GFE, and ſo of the reſt, having each three of thoſe equal arches for its baſe, will be alſo equal, (*by the* 27. 3.) Therefore the ſides and angles of the Pentagon DHEFG are equal.

PROPOSITION XII.

A PROBLEM.

To deſcribe a Pentagon about a Circle



INſcribe a regular Pentagon ABCDE in the circle, (*by the* 11,) and having drawn tangents through the points A, B, C, D, E, (*by the* 17. 3.) you will have deſcrib'd a regular Pentagon about the circle. Draw the lines *FA*, *FG*, *FE*, *FH*, *FD*.

Demonſtration.

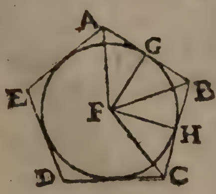
The tangents GE and GA are equal, *(by coroll.*

roll. 2. *of the* 36. 3.) as alſo EH and HD: the lines FA and FE are alſo equal (*by the definit. of a circle*;) therefore (*by the* 8. 1) the triangles FGA and FGE are equal in all reſpects; and the angles AFG and EFG are equal, as alſo the angles EFH and DFH. And becauſe (*by the* 27. 3.) the angles EFA and EFD are equal, their halves EFH and EFG will be equal; and (*by the* 26. 1.) the triangles EFH and EFG will be equal in all reſpects, and the ſides EG and EH will be alſo equal. After the ſame manner I can demonſtrate all the ſides to be divided into two equal parts; and conſequently, ſince the lines GE and GA are equal, GH and GI will be alſo equal. Further, the angles G and H being double the angles FGE, and FHE, are alſo equal. Therefore we have deſcrib'd a regular Pentagon about the circle.

PROPOSITION XIII.

A PROBLEM.

To inſcribe a circle in a regular Pentagon.



TO inſcribe a circle in the regular Pentagon ABCDE, divide the angles A and B into two equal parts by the lines AF and BF, which concur at the point

point F. Then drawing the line FG perpendicular to AB, deſcribe a circle from the center F at the diſtance FG. I ſay it will touch all the other ſides; that is to ſay, having drawn FH perpendicular to BC, FH and FG will be equal.

Demonſtration.

Since the equal angles A and B were divided into two equal parts, their halves GAF and GBF will be equal: And ſince the angles at G are right angles, the triangles AFG and BFG will be equal in all reſpects, *(by the* 26. 1.) therefore the lines AG and GB are equal.

Further, I may prove the lines BG and BH, as alſo the lines FG and FH, to be equal; and the ſides AB and BC of a regular pentagon being equal, the lines BH and HC will be equal, and conſequently, the angles at the point H being alſo right angles, and equal, the triangles BFH and HFC will be equal in all reſpects, and the angles FBH and FCH will be equal. And ſince the angles B and C are equal, the angles FBH will be half the angle C. So paſſing from one to the other I will demonſtrate, that all the perdendiculars FG and FH, and the reſt, are equal.

PROP.

PROPOSITION XIV.

A Problem.

To describe a circle about a regular Pentagon.

TO deſcribe a circle about the regular Pentagon ABCDE, divide equally two of its ſides AB and BC at G and H, and draw the perpendiculars GF and HF. The circle drawn from the center *F*, at the diſtance FA, will paſs through B, C, D, E.

Demonſtration.

Suppoſe the circle deſcrib'd, it is evident (*by the* 1. 3.) that having divided the line AB in the middle in G, and drawn the perpendicular GF, the center of the circle muſt be in that perpendicular: It is alſo in HF; therefore it is at the point F.

The USE.

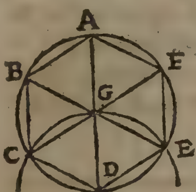
'These Propoſitions are ſolely uſeful for the 'compoſing the table of Sines, and drawing the 'platforms of Gittadels, for their ordinary fi'gures are Pentagons. Obſerve alſo that theſe 'methods of deſcribing Pentagons about a cir'cle, may be apply'd likewiſe to other Polygons.

'gons. But in my book of Military Archite-'cture, I have ſhewn another way of inſcribing 'a regular Pentagon in a circle.

PROPOSITION XV.

A PROBLEM.

To inſcribe a regular Hexagon in a Circle.



TO inſcribe a regular Hexagon in the circle ABC DEF, draw the diameter AD, and fixing the foot of the compaſs at the point D deſcribe a circle at the diſtance of DG: then draw the diameters EGB, and CGF; and the lines AB, AF, FE, and the reſt. *Demonſtration.*

Tis evident, that the triangles CDG, and DGE, are equilateral; therefore the angles CGD, DGE, and thoſe oppos'd to them at the top BGA, and AGF, are each of them the third part of two right angles; that is to ſay, contain 60 degrees. Now all the angles that can be made about the ſame point are equal to four right angles, that is to ſay, 360 degrees. Therefore taking away four times 60, that is 240, from 360, there will remain 120 for BGC and FGE; which therefore each contain

60

60 degrees. Therefore all the angles at the center being equal, all the arches and all the sides will be equal; and every angle as A, B, C, *&c.* will be compounded of two angles of 60 degrees each, that is, 120 degrees, and therefore will be equal.

Coroll. The ſide of a Hexagon is equal to the ſemidiameter.

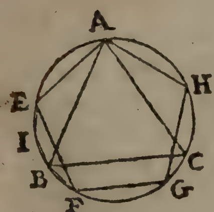
The U S E.

'Becauſe the ſide of a Hexagon is the baſe of 'an arch of 60 degrees, and is equal to the ſe-'midiameter, its half will be the ſine of 30; by 'which ſine we begin the table of Sines. *Euclid* 'treats of Hexagons in the laſt Book of his *Ele-'ments.*

PROPOSITION XVI.

A Problem.

To inſcribe a regular Pentedecagon in a circle.



INſcribe in the circle an equilateral triangle ABC (*by the* 2) and a regular pentagon (*by the* 11.) ſo that the angles may meet at the point A. The lines BF, BI, and IE, will be the ſides of a Pentedecagon: and if you inſcribe in the other arches

arches lines equal to BF and BI, you will compleat the Polygon.

Demonſtration.

Since the line AB is the ſide of an equilateral triangle, the arch AEB will be the third part of the whole circle, that is, five fifteenths. But the arch AE being the fifth part, will contain three fifteenths; therefore the arch EB contains two: and if you divide it in the middle at the point I, each part will be a fifteenth.

The USE.

'This Propoſition ſerves only to open the 'way to other Polygons. We have in the *Com-* '*paſs of Proportion* ſome moſt eaſie methods of 'inſcribing all ordinary Polygons, but they are 'grounded on this here. For it would be im- 'poſſible to mark Polygons upon that inſtru- 'ment, if their ſides were not firſt found by 'this, or other like Propoſitions.

THE

'par'd with the leſs; whether it do really con-
'tain the leſs, or not.

'A Part taken in general is ordinarily divided 'into (that which is call'd) an *Aliquot* part, and 'an *Aliquant* part.

1. An *Aliquot* part (which alone *Euclid* defines in this Book) is a magnitude of a magnitude, a leſs of a greater, when it exactly meaſures the greater. 'That is to ſay, 'tis a leſſer 'quantity compar'd with a greater, which pre'ciſely meaſures the greater. As a line two foot 'long taken three times, is equal to a line of ſix 'feet in length.

2. A *Multiple* is a magnitude of a magnitude, a greater of a leſs, when it is exactly meaſur'd by the leſs. " That is to ſay, a Multi'tiple is a greater quantity compar'd with a 'leſs, which it contains exactly ſo many times. 'For example, a line ſix foot long is triple a 'line two foot long.

An *Aliquant* part, is a leſſer quantity compar'd with a greater, which it does not exactly meaſure. As a line of four feet in length is an aliquant part of a line ten foot long. " In a 'word, An Aliquot part ſo many times repeat'ed will equal the whole: but an Aliquant part, 'though it contains ſuch a quantity of the 'whole, yet repeated as you pleaſe will never 'exactly equal, but either come ſhort of, or ex'ceed, the whole.

Equi-

12,	4;	6,	2.
A,	B;	C,	D

Equimultiples are magnitudes which equally contain their aliquot parts. ' That is to say, so many times. As ' for example, if A contains ' B as many times as C contains D, A and C will ' be equimultiples of B and D.

3. * *Proportion* is a respect of one magnitude to another of the same kind.

* *Græc.* λόγ☉. *Gall* Raison.

4. *Quanties* are said to have a certain proportion to each other, when being mutiply'd they can exceed each other. " For which rea-' son they ought to be of the same kind. For ' indeed a line has no proportion to a superfi-' cies; because a line taken *Mathematically* is ' consider'd without any breadth at all; and ' therefore multiply'd as much as you please ' will never give any breadth, which yet a su-' perficies contains.

' For as much as Proportion is a respect or ' relation founded upon quantity, it ought to have two terms. That which some Philoso-' phers would call the *Fundamentum*, or Foun-' dation, *Mathematicians* name the *Antecedent*, ' and the *Term* is call'd by them the *Consequent*. ' As if we were to compare the quantity A with ' the quantity B, that respect or proportion ' would have for the Antecedent the quantity ' A, and for the consequent the quantity B.

O 2 ' On

'On the contrary, if B be compar'd with A, that 'proportion of B to A would have B for the an-'tecedent, and A for the consequent.

'Proportion, or the respect of one quantity to 'another, is divided into Rational Proportion, 'and Irrational.

'Rational Proportion is the respect of one 'quantity to another, which is commensurable 'to it, that is, when both the quantities have the 'same common measure, by which both may be 'exactly measur'd. As the proportion of a line 'four foot long to another that is six, is rational, 'because a line two foot long may exactly mea-'sure both. A when this happens, these quan-'tities have the same proportion as one number 'to another. For example, since the line two 'foot long, which is their common measure, is 'found twice in the four-foot line, and thrice 'in that which is six foot long; the first has the 'same proportion to the second, as 2 to 3.

'Irrational Proportion is betwixt two quan-'tities of the same kind, which are incommen-'surable, *i. e.* have no common measure. As the 'proportion of the side of a square to its diago-'nal. For there is no measure so small, as will 'precisely measure both.

'Four quantities will be proportionals, when 'the proportion of the first to the second, is the 'same with, or like to, that of the third to the 'fourth; so that, to speak properly, Proporti-'onality

'onality is a ſimilitude of proportions. But it is 'no eaſie matter to underſtand in what conſiſts 'this ſimilitude of proportions; that is to ſay, 'how two reſpects of relations may be alike. 'For *Euclid* has not given us a juſt definition 'thereof, or ſuch an one as might explain the 'nature of the thing, but contented himſelf to 'ſet down ſome marks or ſigns, by which it 'may be known, whether or no quantities have 'the ſame proportion. And 'tis the obſcurity of 'this definition, which has render'd the whole 'Book ſo difficult to be underſtood; which de- 'fect therefore I ſhall endeavour to ſupply.

5. *Euclid* makes four magnitudes to have the 'ſame proportion, when taking the Equimulti- 'ples of the firſt and the third, and likewiſe the 'Equimultiples of the ſecond and the fourth, 'according to any multiplication whatſoever: 'if the multiple of the firſt exceed that of the 'ſecond, the multiple of the third will alſo ex- 'ceed that of the fourth; and if it be equal to, 'or leſs than the ſecond, the third will be equal 'to or leſs than the fourth. In ſuch a caſe the 'firſt has the ſame proportion to the ſecond, as 'the third to the fourth.

A,	B	::	C,	D
2,	4	::	3,	6
E,	F	::	G,	H
10,	8	::	15,	12
K,	L	::	M,	N
8,	8	::	12,	12
O,	P	::	Q	R
6,	16	::	9,	24

'As for example, if four 'magnitudes were propos'd, 'A, B, C, D; having taken 'the Equimultiples of A 'and C, as their quintuples 'E and G; and F and H the 'doubles of B and D; In 'like manner taking K and 'M the quadruples of A and 'C, and L and N the doubles of B and D; A-'gain taking O and Q the triples of A and C, 'and P and R the quadruples of B and D; Be-'cause E being greater than F, G is greater 'than H; and K being equal to L, M is equal 'to N; and lastly, O being less than P, Q is 'less than R: therefore A will have the same 'proportion to B, as C to D. But methinks *Eu-'clid* ought to have demonstrated this Proposi-'tion, it being too perplex'd and obscure to pass 'for a Principle.

'To explain aright what Proportionality is, 'or how four magnitudes may be in the same 'proportion; though it may be sufficient to say 'in general, that the first ought to be a like 'part, or a like whole in respect of the second; 'as the third is, compar'd with the fourth: yet 'because this definition agrees not to the pro-'portion of Equality, I shall give a more gene-'neral one; and to make it the more intelligible, 'explain first what is a similar or like Aliquot 'part.

'Similar

'Similar Aliquot parts, then are ſuch as are 'contain'd in their wholes as many times one 'as the other: as 3 in reſpect of 9, and 2 in re-'ſpect of 6, are ſimilar aliquot parts; becauſe they 'are each contain'd three times in their reſpe-'ctive wholes.

'The firſt quantity will have the ſame propor-'tion to the ſecond, as the third to the fourth, 'if the firſt contains ſo many times ſuch aliquot 'parts of the ſecond, as the third contains the 'like aliquot parts of the fourth. As

A,B,C,D,

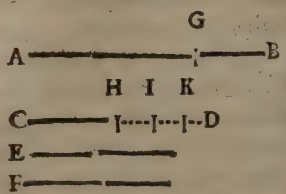
'if A contains the hundredth, thou-'ſandth, or hundred-thouſandth 'part of B, as oft as C contains the hundredth, 'thouſandth, or hundred-thouſandth part of D; '(and the like may be ſaid of all other aliquot 'parts imaginable:) there will be the ſame pro-'portion of A to B, as of C to D.

'To render this Definition ſtill more clear, I 'will prove firſt that, if A has the ſame propor-'tion to B as C to D, A will contain the ali-'quot parts of B, as oft as C does the like of D; 'and ſecondly, if A contain the aliquot parts of 'B as oft as C does the like of D, then there 'will be the ſame proportion of A to B, as of 'C to D.

'The firſt point ſeems ſufficiently evident 'from the very notion of the terms: for if A 'contains the tenth part of B once more than an 'hundred times, and C contains the tenth part

‘ of D an hundred times only; the quantity A ‘ will be a greater Whole compar'd to B, than ‘ C compar'd to D; therefore it cannot be compar'd after the ſame manner, the reſpect or re‘ lation being not the ſame.

‘ The ſecond point ſeems more difficult, *viz.* ‘ if a quantity, ſuppoſe AB, contain the aliquot ‘ parts of another, CD, as oft as a third, E, con‘ tains the like of a fourth, F : there will be ‘ the ſame proportion of AB to CD, as of E to ‘ F. But if it be other‘ wiſe, let us ſuppoſe AB to have a greater proportion to CD, ‘ than E has to F; that ‘ is to ſay, that AB is too great to have the ‘ ſame proportion to CD, that E has to F: Therefore a quantity leſs ‘ than AB, as AG, will have the ſame propor‘ tion to CD, as E to F. Divide therefore CD in ‘ the middle in H, and HD in the middle in I, ‘ and ID in the middle in K; continuing the ‘ like diviſion till you arrive at an aliquot part ‘ of CD leſs than GB, which I will ſuppoſe to ‘ be KD.



Demonſtration.

‘ Since there is the ſame proportion of AG ‘ to CD, as of E to F; AG will contain KD, ‘ an aliquot part of CD, as oft as E contains the like

'like aliquot part of *F.* Now AB will contain 'KD once more than E contains the like aliquot 'part of *F*; which is contrary to the ſuppoſiti- 'on.

6. The firſt Quantity is ſaid to have a greater proportion to the ſecond, than the third to the fourth, when the firſt contains a certain aliquot part of the ſecond oftner than the third contains the like aliquot part of the fourth: As '101 has a greater proportion to 10 than 200 'to 20, becauſe 101 contains the tenth part of '10, that is, 1, once above a hundred times; and '200 contains the tenth part of 20, *i.e.* 2, a 'hundred times only.

7. Magnitudes or Quantities having the ſame proportion, are call'd proportionals.

8. * Proportionality, or Analogy, is a ſimilitude of Proportions or Reſpects.

* Ἀναλογία. *Eucl.*

9. In each proportionality are required at the leaſt three terms. " *For* that there may be 'a ſimilitude of proportions, there muſt be two 'of them: and every proportion having two 'tems, an Antecedent and a Conſequent, there 'ſeems to be a neceſſity of four terms; as when 'we ſay, that A has the ſame proportion to B 'as C to D; but becauſe the conſequent of the 'firſt proportion may be the antecedent of the 'ſecond, three terms may ſuffice; as when A is 'ſaid to have the ſame proportion to B as B 'to C.

10. Mag-

10. Magnitudes are ſaid to be continually proportionals, when the intermediate terms are taken twice, *i. e.* both as antecedents and conſequents. “ As if there be the ſame proportion ‘ of A to B, as of B to C, and of C to D.

11. In that caſe A will have the duplicate proportion to C, and the triplicate to D, of what it has to B.

‘ But here it is to be obſerv’d, that there is a ‘ great deal of difference between double pro- ‘ portion, and duplicate. We ſay that the pro- ‘ portion of four to two is double, becauſe four ‘ is the double of two; the number two giving ‘ the name to the proportion, or rather to the ‘ antecedent thereof. Accordingly double, tri- ‘ ple, quadruple, quintuple, *&c.* are denomina- ‘ tions drawn from the numbers two, three, four, ‘ five, *&c.* compar’d with unity; which I in- ‘ ſtance in, becauſe we more eaſily conceive the ‘ proportion, the leſs are its terms. But, as I ‘ ſaid, theſe denominations do rather affect the ‘ antecedents, than the proportions themſelves; ‘ for we call that double or triple proportion, ‘ whoſe antecedent is double or triple its con- ‘ ſequent. But by duplicate proportion we un- ‘ derſtand ſuch an one, as is compounded of two ‘ ſimilar proportions. As, if there be the ſame ‘ proportion of two to four, as of four to eight: ‘ the proportion of two to eight being com- ‘ pounded of the proportion of two to four, and that

'that of four to eight, which are ſimilar an equal, will be the duplicate of each of them. So three to twenty ſeven is the duplicate proportion of that of three to nine. The proportion of two to four is call'd the ſub-double, becauſe two is the half of four; but that of two to eight is the duplicate of the ſub-double: which is as much as to ſay, that two is the half of half of eight, as three is the third part of the third part of twenty ſeven; where you may obſerve, that the Denominators $\frac{1}{2}$ and $\frac{1}{3}$ are taken twice.

'In like manner the proportion of eight to two is a duplicate proportion of that of eight to four, becauſe eight is the double of four, but it is the double of the double of two. If there be four terms in continual proportion, the proportion of the firſt to the laſt is a triplicate of that of the firſt to the ſecond; as in theſe four numbers, Two, Four, Eight, Sixteen; the proportion of two to ſixteen is a triplicate of that of two to four, becauſe two is the half of the half of the half of ſixteen. So alſo the proportion of ſixteen to two is a triplicate of that of ſixteen to eight, becauſe ſixteen is the double of the double of the double of two.

12. Antecedents to antecedents, and conſequents to conſequents, are call'd Homologous magnitudes. As if there be the ſame proportion of

'of A to B, as of C to D; A and C, B and D 'are homologous.'

'The following Definitions explain the divers 'manners of arguing from proportionals: for the 'demonstration of which this Book was princi-'pally compos'd.

13. Alternate proportion is when we compare the antecedent of the one with the antecedent of the other, and the consequent of the one with the consequent of the other. "As for 'example, if because there is the same propor-'tion of A to B, as of C to D, I infer, that there 'is the same proportion of A to C as of B to D. 'This manner of argumentation holds only when 'all the four terms are of the same *species* or 'kind; *i.e.* either all lines, or all superficies's, 'or all solids. 'Tis demonstrated *Prop.* 16.

14.*Inverted proportion is the comparing of 'the consequents with the antecedents.

Ἀνάπαλιν Eucl. Converse *Gall.*

'As, if because there is the same proportion 'of A to B, as of C to D, I conclude that there 'is the same proportion of B to A, as of D to 'C. *Coroll. of Propos.* 16.

15. Composition of proportion is the comparing of the antecedent and the consequent taken together, with the consequent alone. As 'if, because there is the same proportion of A 'to B, as of C to D, I conclude that there is 'the same proportion of A and B to B, as of C 'and D to D. *Prop.* 18.

16. Di-

16. Diviſion of Proportion is the comparing of the Exceſs of the antecedent above the conſequent to the ſame conſequent. " As, if there ' be the ſame proportion of AB to B, as of CD ' to D; from thence I infer, that there is the ' the ſame proportion of A to B, as of C to D. *Propoſition* 17.

17. Converſion of Proportion is the comparing of the antecedent with the Difference of the Terms: " As, if there be the ſame pro- ' portion of AB to B, as of CD to D. I thence ' conclude, that there is the ſame proportion of ' AB to A, as of CD to C. *Coroll. of Prop.* 18.

18. Proportion of Equality is the comparing of the extream magnitudes, and omitting thoſe in the middle. " As if, there be- ' ing the ſame proportion of A to ' to B, as of E to F; and of B to ' C, as of F to G; and of C to D, ' as of G to H; I infer, that there is the ſame ' proportion of A to D, as of E to H.

A.B.C.D.
E.F.G.H.

19. Proportionality or Equality orderly plac'd, is that in which the terms are compar'd together in the ſame order. " As in the fore- ' going example. *Prop.* 22.

20. Proportionality of Equality diſorderly plac'd, is that in which the terms are compar'd in a different order. ', As if, there being the ' ſame proportion of A to B, as of G to H; and ' of B to C, as of F to G; and of C to D, as of

E

'E to F; I conclude that there is the ſame pro-
'portion of A to D, as of E to H. *Prop.* 23.

'See in ſhort all the different manners of ar-
'gumentation by Proportion.

'As A to B, ſo C to D; therefore
'By Alternate proportion: as A to C, ſo B
'to D:
'Inverted: As B to A, ſo D to C
'Compoſition. As AB to B, ſo CD to D.
'Diviſion. If as AB to B, ſo CD to D: then
As A to B, ſo C to D.
'Converſion. As AB to A, ſo CD to C.
Orderly Equality. If as A to B, ſo C to D; and
as B to E, ſo D to F: then
as A to E, ſo C to F.
Diſorderly Equality. If as A to B, ſo D to F, and
as B to E, ſo C to D: then
as A to E, ſo C to F.

'*Euclid's* fifth Book contains but 25 propoſi-
'tions, to which nine more have ſince been ad-
'ded, and are commonly receiv'd. And the firſt
'ſix in *Euclid*, ſerving only for the proof of
'thoſe that follow by the method of *Equimul-*
'*tiples*, ſince I intend not to make uſe of that me-
'thod, I ſhall wholly omit; begining with the
'Seventh, without changing either the order or
'number of the propoſitions.

Demands

Demands, or Suppositions.

Three quantities A, B, C, being propos'd, it is requir'd to be granted, that there is a fourth possible, to which the quantity C has the same proportion as A to B.

PROPOSITION VII.

A Theorem

Equal quantities have the same proportion to another quantity, and another quantity has the same proportion to quantities that are equal.

A, 8.	
	C. 4.
B, 8.	

If the quantities A and B be equal, they will have the same proportion to the third C.

Demonstration.

If one of the two, suppose A, had a greater proportion to C than B: A would contain any aliquot part of C, oftner than B could contain the same; and consequently A would be greater than B, which is contrary to what was suppos'd.

Again, I say, if the quantities A and B be equal, the quantity C will have the same proportion to A as to B.

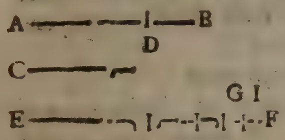
Demonſtration.

If the quantity C had a greater proportion to A, than to B, it would contain a certain aliquot part of A, oftner than the like part of B; which part therefore of A muſt be leſs than the like aliquot part of B. and conſequently the quantity A would be leſs than B, which is contrary to the ſuppoſition.

PROPOSITION VIII.

A Theorem.

The greater of two quantities has a greater proportion to the ſame, than the leſs; and the ſame quantity has a leſſer proportion to the greater, than to the leſs.



Suppoſe the quantities AB and C be compar'd with the ſame EF, and that AB exceeds C. I ſay, that AB will have a greater proportion to EF, than C will have to the ſame. Cut AD equal to C, and divide EF in the middle, and again one half in the middle, and ſo on, till you come to an aliquot part of EF leſs than DB, as GF.

Demon-

Demonstration.

AD and C being equal, AD has the ſame proportion to EF, as C to EF, (*by the* 7.) and therefore AD will contain GF an aliquot part of EF, as oft as C will contain the ſame, (*by defin.* 5.) But AB contains the ſame aliquot part once more than AD, DB being greater than GF; therefore (*by defin.* 6.) the proportion of AB to EF is greater than that of C to the ſame EF.

Secondly, I ſay, that EF has a leſs proportion to AB, than to C. Take a certain aliquot part, as the fourth, of C, as oft as you can in EF, which ſuppoſe to be five times; either there will remain ſomething of the quantity EF, or nothing; if nothing remain, it is evident, that five times the fourth part of AB making a greater line than ſo many times the fourth part of C, the fourth part of AB could not be five times contain'd in EF. But if the fourth part of C taken five times reach no farther than G, the fourth part of AB taken ſo many times will proceed either as far as F, or to I, ſomething ſhort of F. If it reach as far as F, EF will have the ſame proportion to AB as EG to C. But (*by the preceding part*) EF has a greater proportion to C, than EG to the ſame C; therefore EF has greater proportion to C, than to AB. But if the fourth part of AB reach no farther than I, EI will have the ſame proportion to AB as EG to

C. But EI has a greater proportion to C, than EG to C; therefore EF, greater than EI, has a greater proportion to C, than the ſame EF to AB.

PROPOSITION IX.

A Theorem.

Quantities that have the ſame proportion to another quantity, or to whom another quantity has the ſame proportion, are equal.

A,	B,	C,
12.	12	6

If the quantities A and B have the ſame proportion to a third quantity C, I ſay, A and B are equal.

Demonſtration.

If one of the two, *v g.* A, were greater than B, it would have a greater proportion to the quantity C, (*by the* 8.) which is contrary to the ſuppoſition.

Secondly, if the quantity C has the ſame proportion to the quantities A and B; I ſay A and B are equal. For if A were greater than B, C would have a greater proportion to the quantity B, than to A, (*by the* 8.) which is alſo contrary to the ſuppoſition.

PRO-

PROPOSITION X.

A THEOREM.

The quantity that has the greater proportion to an other, is the greater quantity; and that the leſſer, to which that other quantity has the greater proportion.

A, B, C,

IF the quantity A has a greater proportion to the quantity C, than B to the ſame C, I ſay, A is greater than B. For if A and B were equal, they would have both the ſame proportion to C; and if A were leſs than B, B would have a greater proportion to C, than A to the ſame C; both which are contrary to the ſuppoſition.

Secondly, if C has a greater proportion to B than to A, I ſay that A will be greater than B. For if A and B were equal, C would have the ſame proportion to both, [*by the* 7.) And if A were leſs than B, C would have a leſs proportion to B than to A, (*by the* 8.) both which are contrary to what was ſuppos'd.

PROPOSITION XI.

A THEOREM.

Proportions that are equal one to another, are alſo equal amongſt themſelves.

A,	B;	C,	D;	E	F.
4,	2;	8,	4;	6,	3.

IF A has the ſame proportion to B, as C to D; and C the ſame proportion to D, as E to F; I ſay, A will have the ſame proportion to B, as E to F.

Demonſtration.

Since A has the ſame proportion to B, as C to D; A will contain any certain aliquot part of B, as oft as C contains the like aliquot part of D, (*by defin.* 5.) And in like manner, as oft as C contains that aliquot part of D, ſo oft will E contain the like aliquot part of F. So that as oft as A contains any certain aliquot part of B, ſo many times alſo will E contain the like aliquot part of F. Therefore A has the ſame proportion to B, as E to F.

PROP.

PROPOSITION. XII.

A THEOREM.

If many quantities be proportional, one Antecedent will have the ſame proportion to his conſequent, as all the Antecedents taken together to all the Conſequents.

A.	B.
3	12,
C,	D.
2,	8.

IF A has the ſame proportion to B, as C to D; I ſay that A and C taken together will have the ſame proportion to B and D, as A to B.

Demonſtration.

Since A has the ſame proportion to B, as C to D; the quantity A will contain any certain aliquot part of B, as oft as C contains the like aliquor part of D, (*by defin.* 5) ſuppoſe the fourth part. Now the fourth part of B and the fourth part of D, make the fourth part of BD; and accordingly AC will contain the fourth part of BD, as oft as A contains the fourth part of B; and the like may be ſaid of any other aliquot parts. Therefore A has the ſame proportion to B, as AC to be BD.

PROPOSITION XIII.

A THEOREM.

If of two equal proportions one is greater than a third, the other will be so likewise.

| A, B : C, D : E, F |

IF A has the ſame proportion to B, as C to D; but a greater proportion to B, than E to F: I ſay, that C alſo will have a greater proportion to D, than E to F.

Demonſtration.

Since A has a greater proportion to B than E to F, A will contain a certain aliquot part of B, oftner than E contains the like aliquot part of F, (*by defin.* 6.) but C contains a like aliquot part of D, as oft as A contains that of B; becauſe A has the ſame proportion to B, as C to D: and therefore C contains a certain aliquot part of D, oftner than E contains the like aliquot part of F; and conſequently, C has, a greater proportion to D, than E to F, (*by def.* 6.)

PROP.

PROPOSITION XIV.

A THEOREM.

If the firſt quantity has the ſame proportion to the ſecond, as the third to the fourth; according as the firſt is greater, or equal, or leſs, than the third, the ſecond will be greater, or equal, or leſs, than the fourth.

A, B: C, D.

IF A has the ſame proportion to B, as C to D; I ſay firſt, if A be greater than C, B will be alſo greater than D.

Demonſtration.

Since A is greater than C, A will have a greater proportion to B, than C to the ſame B, (*by the* 8.) But there is the ſame proportion of A to B, as of C to D: therefore C has a greater proportion to D, than C to B, and conſequently (*by the* 10) B is greater than D.

Secondly, if A be equal to C, B will be alſo equal to D.

Demonſtration.

Since A and C are equal, there will be the ſame proportion of A to B, as of C to the ſame B, (*by the* 7.) But as A to B, ſo C to D; therefore C has the ſame proportion to B, as the ſame C to D, and conſequently B and D are equal, (*by the* 9.)

Thirdly, if A be leſs than C, B will alſo be leſs than D. *Demonſtration.*

Since A is leſs than C, A will have a leſs proportion to B, than C to the ſame B, (*by the* 8.) But as A is to B, ſo C to D: therefore C will have a leſs proportion to D, than the ſame C to B, and conſequently B will be leſs than D, (*by the* 10.)

PROPOSITION XV.

A Theorem.

Equimultiples, and ſimiliar aliquot parts, are in the ſame proportion.

A, B;	C, D,
2, 3;	6, 9.
E, 2;	H, 3,
F, 2;	I, 3,
G, 2;	K, 3,

If the quantities C and D be the Equimultiples of A and B, their aliquot parts, A will have the ſame proportion to B, as C to D. Divide the quantity C into parts equal to A, *v. g.* E, F, G, and the quantity D into parts equal to B. Becauſe C and D are the equimultiples of A and B, there will be as many parts of one, as of other other. Let the parts of D therefore be H, I, K. *Demonſtration.*

E has the ſame proportion to H, and F to I, and G to K, as A to B, becauſe they are all e-qual.

qual. Therefore *(by the* 12.*)* as A to B, ſo E, F, G, to H, I, K, *i. e.* ſo C to D,

Coroll. The ſame numbers of the aliquot parts of two quantities are in the ſame proportion that the quantities are For ſince E has the ſame proportion to H, as C to D; and F to I, as C to D; E and F will have the ſame proportion to H and I, as C to D.

PROPOSITION XVI.

A Theorem.

Alternate Proportion.

If four magnitudes of the ſame kind be proportional, they will be alſo alternatively ſo.

A, B;	C, D,
12, 8;	9, 6

If A has the ſame proportion to B as C to D, and all the four quantities are of the ſame kind, that is, either all Lines, or all Supercifies's, or all Solids; A will have the ſame proportion to C, as B to D. For if not, ſuppoſe A to have a greater proportion to C, than B to D.

Demonſtration.

Since tis ſuppos'd, that A has a greater proportion to C than B to D, the quantity A will contain a certain aliquot part of C, *v. g.* a third part

part, oftner than B contains a third part of D. Let A therefore contain a third part of C four times, but B the third part of D only three times; having then divided A into four parts, each will contain one third part of C; but B being divided into four parts, they will not contain each of them a third part of D; therefore three fourths of A will contain three thirds of C, that is, tho whole quantity of C; but three fourths of B will not contain three thirds, or the whole quantity of D. But on the contrary, ſince there is the ſame proportion of A to B, as of C to D, there will be alſo the ſame proportion of three fourths of A to three fourths of B, as of C to D, (*by the coroll. of the* 15) and (*by the* 14) if three fourths of A be equal to C, three fourths of B will be equal to D; therefore A cannot have a greater proportion to C, than B to D.

A *LEMMA*.

If the firſt have the ſame proportion to the ſecond, as the third to the fourth; any aliquot part of the firſt will have the ſame proportion to the ſecond, as the like aliquot part of the third to the fourth.

16,	3;	32,	6,
A,	B;	C,	D,
E,		F.	
4,		8.	

If A has the ſame proportion to B, as C to D; and E be an aliquot part of A, and F a like aliquot part of C,

C: I ſay, that E will have the ſame proportion to B, as F to D.

Demonſtration.

If E had a greater proportion to B than F to D, E would contain a certain aliquot part of B, oftner than F contains the like aliquot part of D; and conſequently, E taken twice, thrice, or four tims, would contain an aliquot part of B, oftner than F taken twice, thrice or four times contain'd the like aliquot part of D. But E taken four times is equal to A; and likewiſe F taken four times is equal to C; therefore A would contain an aliquot part of B oftner than C contain'd the like aliquot part of D, and by conſequence A would have a greater proportion to B than C to D; which is contrary to the ſuppoſition.

A COROLLARY,

" which *Euclid* places after his 4*th* Propoſition,

Inverted Proportion

If the firſt has the ſame proportion to the ſecond, as the third to the fourth; the ſecond will have the ſame proportion to the firſt, as the fourth to the third.

A, B;	C, D,
4, 8;	12, 24,
E,	F,
1,	3.

IF A has the ſame proportion to B as C to D, B will have the ſame proportion to A as D to C.

De-

Demonſtration.

If B had a greater proportion to A than D to C, B would contain an aliquot part of A, ſuppoſe a fourth of E,oftner than D contain'd F the fourth part of C. Let us ſuppoſe then that B contains eight times the quantity E; D muſt contain but ſeven times the quantity F. Now ſince A has the ſame proportion to B as C to D, E will have the ſame proportion to B as F to D, (*by the preceding Lemma,*) and (*by the* 15) E taken eight times will have the ſame proportion to B, as F taken eight times to D; but E taken eight times is contain'd in B, therefore F taken eight times muſt be contain'd in D, notwithſtanding what was ſhewn to the contrary; therefore B cannot have a greater proportion to A, than D to C.

The USE.

'The Followers of *Averroes* ſeem to have 'made uſe of a manner of argumentation not 'much unlike this, to prove that *the world had* '*exiſted from Eternity*; urging, that there is the 'ſame proportion between an eternal act of the 'will of God,and the eternal production of the 'world, as between a temporal act, and a temporal effect; therefore by a kind of Alternation, there is the ſame proportion of a tempo'ral act of the will, *i. e.* an act beginning in time, 'to an eternal effect; as of an eternal will to a 'temporal effect. Now tis evident, that the will

'will, or an act of the will that begins in time, 'cannot produce an eternal effect; therefore 'the eternal act of God could not produce an 'Effect in time. But this argument is faulty in 'two reſpects; firſt, in that it ſuppoſes it poſſi-'ble for an act of the divine will to begin in 'time; and ſecondly, in that it is drawn from 'Alternate proportion, though the terms be of 'a different kind or *ſpecies.*

PROPOSITION XVII.

A THEOREM.

The Diviſion of Proportion.

If compounded quantities be proportional, they will be ſo likewiſe being divided.

AB,	B;	CD,	D,
8,	3;	16,	6,
A,	B;	C,	D,
5,	3;	10,	6.

IF AB has the ſame proportion to B as CD to D, A will have the ſame proportion to B, as C to D.

Demonſtration.

Since AB is ſuppos'd to have the ſame proportion to B as CD to D, AB will contain a certain aliquot part of B, as oft as CD contains the like aliquot part of D. Now that aliquot part muſt be found as oft in B, as the like aliquot part is found

found in D. Therefore taking away B from AB, and D from CD, A will contain as many aliquot parts of B, as C contains the like of D, and consequently A will have the same proportion to B, as C to D.

PROPOSITION XVIII.

A Theorem.

The Composition of Proportion.

If quantities, being divided, be proportional, they will be so likewise when compounded.

A,	B;	C,	D,
5.	3;	10,	6.
AB,	B;	CD,	D,
8,	3;	16,	6,

If A has the same proportion to B as C to D, AB also will have the same proportion to B as CD to D.

Demonstration.

Since A is suppos'd to have the same proportion to B as C to D, A will contain any aliquot part of B, as oft as C contains the like aliquot part of D. Now the quantity B contains any of its own aliquot parts, as oft as D contains the like of his; therefore adding B to A, and D to C, AB will contain any aliquot part of B as oft as CD contains the like aliquot part of D, and consequently (*by defin.* 5.) AB will have the same proportion to B as CD to D.

A

A COROLLARY.

The Converſion of Proportion:

If AB has the ſame proportion to B as CD to D, then AB will have the ſame proportion to A as CD to C. For (*by the preceding*) A has the ſame proportion to B, as C to D: and (*by the Coroll. of the* 16.) B will have the ſame proportion to A, as D to C; and therefore compounding them, AB will have the ſame proportion to A, as CD to C.

The USE.

" We have frequent uſe of this manner of ar-
" gumentation in almoſt all parts of the *Mathe-*
" *maticks.*

PROPOSITION XIX.

A THEOREM.

If the Wholes be in the ſame proportion, as the parts that are taken away from them, the Remainders will be alſo in the ſame proportion.

AB, CD;	B, D,		
16, 8;	4, 2,		
A, C;	AB, CD,		
2, 6;	16, 8,		

IF the quantity AB has the ſame proportion to CD, as the part B to the part D: I ſay, A will have

have the same proportion to C, as AB to CD.

Demonstration.

AB is suppos'd to have the same proportion to CD, as B to D: therefore alternatively (*according to the* 16.) AB has the same proportion to B, as CD to D; and by conversion of proportion, AB will have the same proportion to A, as CD to C; and again alternatively, there will be the same proportion of AB to CD, as of A to C.

The USE.

'This Proposition is commonly made use of 'in the rule of Fellowship For instead of work'ing by the rule of Three for every particular 'Associate or Partner, having done it for the 'rest, to the last they assign the Remainder of 'the Gain; supposing that if there be the same 'proportion of the whole sum of all the Prin'cipals to the whole Gain, as of the Principal 'of one Associate to his part of the Gain: there 'will be also the same proportion of the Princi'pal that remains to the Remainder of the 'Gain.

'The 20 and 21 Propositions are not neces'sary.

PROP.

PROPOSITION XXII.

A THEOREM.

The proportion of Equality orderly plac'd.

If divers terms be propos'd, and an equal number of others compar'd with them, ſo that thoſe which anſwer to each other in the ſame order be proportional; the firſts and the laſts will be alſo proportional.

12,	6,	2;	6,	3,	1,
A,	B,	C;	D,	E,	F.

IF the quantities A,B,C and the quantities D,E F, be proportional; that is, if there be the ſame proportion of A to B as of D to E, and of B to C as of E to F; A will alſo have the ſame proportion to C, as D to F.

Demonſtration.

If A has a greater proportion to C than D to F, A will contain an aliquot part, *v. g.* the half of C, oftner than D can contain the half of F. Let us ſuppoſe then the half of C to be contain'd twelve times in A, and the half of F only eleven times in D. Now becauſe B has the ſame proportion to C as E to F, the quantity B will contain the half of C, as oft as E contains the half of F: Suppoſe then thoſe halves to be con-

Q tain'd

tain'd ſix times in each, B and E. A, which contains the half of C twelve times, will have a greater proportion to B, which contains the ſame half of C ſix times, than D, which contains the half F eleven times only, to E, which contains the ſame half of F ſix times; and conſequently A will have a greater proportion to B, than D to E, which is contrary to what was ſuppos'd.

PROPOSITION XXIII.

A PROBLEM.

The proportion of Equality diſorderly plac'd.

If two Orders of terms, be in the ſame proportion, diſorderly plac'd: the firſt and laſt of both Orders will be proportional.

A,	B,	C.	D,	E,	F.	G,
12,	6,	3.	8,	4,	2.	1,

If the quantities A,B,C, and the others D, E, F, equal in number, be in the ſame proportion, diſorderly plac'd, that is, if A has the ſame proportion to B as E to F, and B to C as D to E: A will have the ſame proportion to C, as D to F. Suppoſe B to have the ſame proportion to C, as F to G.

Demon-

Demonſtration.

Since there is the ſame proportion of A to B as of E to F, and of B to C as of F to G; A has the ſame proportion to C, as E to G, *(by the* 22.*)* Further, ſince B has the ſame proportion to C as D to E, and alſo as F to G; D will have the ſame proportion to E, as F to G, *(by the* 11.) and alternatively; *(according to the* 16.) D will have the ſame proportion to F, as E to G. Now we have before prov'd, that as E to G, ſo A to C; therefore, as A to C, ſo D to F.

PROPOSITION XXIV.

A Theorem.

If the firſt quantity has the ſame proportion to the ſecond, as the third to the fourth; and alſo the fifth to the ſecond, as the ſixth to the fourth: the firſt with the fifth will have the ſame proportion to the ſecond, as the third with the ſixth to the fourth.

E,	F.
4,	6.
6, 2;	9, 3,
A, B;	C, D,

If A has the ſame proportion to B as C to D, and E to B as F to D; AE will have the ſame proportion to B, as CF to D.

Demonſtration.

Since A has the ſame proportion to B as C to D,

D, A will contain any aliquot part of B, as oft as C contains the like aliquot part of D, (*by defin.* 5) In like manner, E will contain the ſame aliquot part of B, as oft as F contains the like of D; ſo that A and E will contain any aliquot part of B, as oft as C and F contain the like aliquot part of D: therefore AE will have the ſame proportion to B, as CF to D.

PROPOSITION XXV.

A Theorem.

If four magnitudes be proportional, the greateſt and the leaſt will exceed the other two.

B,	D,		
8,	6,		
4, 3;		4, 3,	
A, C;		E, F,	

If the four magnitudes *AB*, *CD*, *E*, *F*, be proportional; and *AB* the greateſt and *F* the leaſt; *AB* and *F* will exceed *CD* and *E*. Since *AB* has the ſame proportion to *CD* as *E* to *F*, and *AB* is ſuppos'd greater than *E*; *CD* will be alſo greater than *F*, (*by the* 14.) Divide therefore *AB* ſo, that the magnitude *A* may be equal to *E*; and *CD* ſo, that the magnitude C may be equal to *F*.

Demonſtration.

Since *AB* has the ſame proportion to *CD* as *A* to C, *B* will alſo have the ſame proportion to *D* as *AB* to *CD*, (*by the* 19.) and *AB* being ſup-

ſuppos'd greater than CD, B will be greater than D. Now if to A and E, which are equal, be added F and C, which are alſo equal, A and F will be equal to C and E, and adding to the two firſt B, which is the greater, and to the two laſt D, which is the leſs, AB and F will be greater than CD and E.

The USE.

By this Propoſition is demonſtrated a pro-
'priety of *Geometrical* proportionality, whereby
'it is diſtinguiſh'd from that which is call'd *Arith-*
'*metical.* For in this latter the two middle terms
'are equal to the two extreams; but in the for-
'mer, (as has been prov'd,) the greateſt and
'the leaſt exceed the two others,

'Tho' the nine following Propoſitions are not
'*Euclids*; yet, becauſe many make uſe of them,
'and quote them as if they were his, I thought
'I ought not to omit them,

PROPOSITION. XXVI.

A THEOREM.

If the first has a greater proportion to the second than the third to the fourth, the fourth will have a greater proportion to the third than the second to the first.

9,	4;	6,	3,
A,	B;	C,	D,
E.			
8.			

IF A has a greater proportion to B than C to D, D will have a greater propo: tion to C than B to A. Suppose E to have the same proportion to B as C to D A will be greater than E, *(by the* 10.)

Demonstration.

There is the same proportion of E to B, as of C to D: therefore *(by the Coroll. of the* 16.) D has the same proportion to C, as B to E. But B has a greater proportion to E than to A, *(by the* 8.) therefore D has a greater proportion to C, than B to A.

PRO.

PROPOSITION. XXVII.

A Theorem.

If the firſt has a greater proportion to the ſecond than the third to the fourth, the firſt will alſo have a greater proportion to the third than the ſecond to the fourth.

9,	4;	6,	3,
A,	B;	C,	D,
E.			
8.			

IF A has a greater proportion to B than C to D, I ſay, that A will have a greater proportion to C than B to D. Let E have the ſame proportion to B, as C to D; in that caſe A muſt be greater than E.

Demonſtration.

E has the ſame proportion to B, as C to D: therefore (*by the* 16.) E has the ſame proportion to C, as B to D. And becauſe A is greater than E, the proportion of A to C will be greater than that of E to C Therefore the proportion of A to C, is greater than that of B to D.

PROPOSITION XXVIII.

A THEOREM.

If the first has a greater proportion to the second than the third to the fourth, the first and the second will have a greater proportion to the second, than the third and the fourth to the fourth.

9, 4;	6, 3,
A, B;	C, D,
E,	
8.	

IF the Proportion of A to B be greater than that of C to D, the Proportion of AB to B will also be greater than that of CD to D. Suppose E to have the same proportion to B, as C to D.

Demonstration.

E has the same proportion to B, as C to D: therefore (*by the* 18.) EB has the same proportion to B, as CD to D. And AB being greater than EB, AB will have a greater proportion to B, than EB to the same B, and consequently than CD to D.

PRO-

PROPOSITION XXIX.

A THEOREM.

If the first with the second has a greater proportion to the second, than the third with the fourth to the fourth; the first will have a greater proportion to the second, than the third to the fourth.

9, 4 ; 6, 3.	
A,B ; C,D,	
E.	
8.	

IF the proportion of AB to B be greater than the proportion of CD to D, the proportion of A to B will be alſo greater than that of C to D. Suppoſe EB to have the ſame proportion to B, as CD to D : EB will be leſs than AB, and E leſs than A:

Demonſtration.

Since EB has the ſame proportion to B, as CD to D ; dividing them, E will have the ſame proportion to B, as C to D, (*by the* 17.) And A being greater than E, the proportion of A to B will be greater than that of E to the ſame B, and conſequently than that of C to D.

PRO-

PROPOSITION XXX.

A THEOREM.

If the proportion of the firſt with the ſecond to the ſecond, be greater than that of the third with the fourth to the fourth; the proportion of the firſt with the ſecond to the firſt, will be leſs than that of the third with the fourth to the third.

A,	B,	C,	D,
9.	4;	6,	3,

If the proportion of AB to B be greater than that of CD to D, the proportion of AB to A will be leſs than that of CD to C.

Demonſtration.

The proportion of AB to B is ſuppos'd to be greater than that of CD to D: therefore (*by the* 29.) the proportion of A to B will be greater than that of C to D; and (*by the* 26.) the proportion of D to C will be greater than that of B to A: therefore being compounded (*by the* 28.) the proportion of CD to C, will be greater than that of AB to A.

PRO.

PROPOSITION XXXI.

A THEOREM

If many quantities are in a greater proportion among themſelves, that an equal number of other quantities, plac'd after the ſame manner; the firſt of the firſt order will be in a greater proportion to the laſt of that order, than the firſt of the ſecond order to the laſt of that.

A,	B,	C,	D,	E,	F.
16,	10,	3,	9,	6,	2,

IF A has a greater proportion to B, than D to E; and a greater proportion to C, than E to F; A will have a greater proportion to C, than D to F.

Demonſtration.

Since A has a greater proportion to B than D to E, A will alſo have a greater proportion to D than B to E; and becauſe B has a greater proportion to C than E to F, B will alſo have a greater proportion to E than C to F. Therefore A will have a greater proportion to D than C to F: and Alternatively *(by the 27.)* A will have a greater proportion to C, than D to F.

PROPOSITION XXXII.

A Theorem.

If many quantities are in greater proportion among themselves, than an equal number of other quantities, plac'd after a different manner; the first of the first order will have a greater proportion to the last of that Order, than the first of the second Order to the last of that.

13,	6,	2,			
A,	C,	E,			
B,		F	H,	I,	K
12,		3	4.	2,	1

IF A has a greater proportion to C than I to K, and C a greater proportion to E than H to I; A will have a greater proportion to E, than H to K. Suppose B to have the same proporteon to C as I to K, and C the same proportion to F as H to I; then A will be greater than *B*, and F than E.

Demonstration.

Since 'tis propos'd that *B* has the same proportion to C as I to K, and C to F as H to I; *B* will have the same proportion to F, as H to K, (*by the* 23.) But A has a greater proportion to F, than *B* to the same F, (*by the* 8.) and the proportion of A to E is greater than that of A to F, because F is greater than E: therefore the proportion of A to E, is greater than that of H to K.

PRO-

PROPOSITION XXXIII.

A Theorem.

If the Whole has a greater proportion to the Whole than the part taken away to the part taken away, the Remainder will have a greater proportion to the Remainder than the Whole to the Whole.

1 3,4 ;	6, 2
A,B ;	C,D.

IF AB has a greater proportion to CD than B to D, A will have a greater proportion to C than AB to CD.

Demonstration.

We suppose that the proportion of AB to CD is greater than that of B to D ; therefore *(by the 27.)* the proportion of AB to B is greater than that of CD to D : and (*by the* 30.) the proportion of AB to A, is less than that of CD to C ; therefore alternatively, the proportion of AB to CD is less than that of A to C:

PRO-

PROPOSITION XXXIV.

A THEOREM.

If two orders of magnitudes be propos'd, and the proportion of the first of the first, to the first of the second, be greater than that of the second to the second; and that greater than that of the third to the third, &c. the whole first order will have a greater proportion to the whole second, than the whole first order except its first magnitude to the whole second order except its first magnitude. But a less proportion than that of the first magnitude of the first order, to the first magnitude of the second; and lastly, a greater proportion, than that of the last magnitude of the first order, to the last of the second.

12, 6, 4,	4, 3, 3,
A, B, C,	E, F, G,

IF the proportion of A to E be greater than that of B to F, and the proportion of B to F greater than that of C to G: I say, that A,B,C, will have a greater proportion to E, F, G, than the proportion of BC to FG.

Demonstration.

'Tis suppos'd the proportion of A to E is greater than that of B to F; and therefore alternatively

ternatively, the proportion of A to B is greater than that of E to F; and by compounding them, the proportion of AB to B greater than that of EF to F; and again alternatively, the proportion of AB to EF greater than that of B to F. And becauſe the proportion of the whole AB to EF is greater than that of the part B to the part F, the proportion of the Remainder A to the Remainder E will be greater than that of the whole AB to the whole EF: In like manner, I may prove the proportion of B to F greater than that of BC to FG, and conſequenty that of A to E much greater than that of BC to FG. Therefore alternatively, the proportion of A to BC is greater than that of E to FG; and compounding them, the proportion of A, B, C, to BC, greater than that of E, F, G, to FG: therefore the proportion of A,B,C, to E, F G, will be greater than that of BC to FG.

Secondly, the proportion of A to E, is greater than that of A,B,C, to E, F, G.

Demonſtration.

I have demonſtrated, that the proportion of the whole A,B,C, to the whole E, F,G, is greater than that of the part BC to the part FG: therefore the proportion of the Remainder A to the Remainder E, will be greater than that of the whole A B,C, to the whole E,F,G, *(by the* 33.)

Thirdly, the proportion of A,B,C, to E,F,G, is greater than that of C to G.

Demon-

Demonstration.

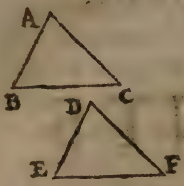
The proportion of A to E is greater than that of B to F; and therefore alternatively, that of A to B is greater than that of E to F; and compounding them, the proportion of AB to B will be greater than that of EF to F; and again alternatively, that of AB to EF greater than that of B to F. But the proportion of B to F is greater than that of C to G, therefore the proportion of AB to EF is greater than that of C to G; and that of AB to C greater than that of EF to G; and therefore by compounding them, the proportion of A, B, C, to C, will be greater than than that of E, F, G, to G; and that of A, B, C, to E, F, G, greater than that of C to G.

THE

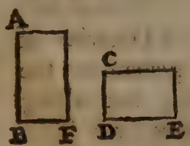
THE SIXTH BOOK OF THE ELEMENTS OF EUCLID.

THis Book begins to apply the Doctrine of Proportions, explain'd in general in the 'preceding, to particular matters; and taking 'its riſe from the moſt ſimple figures, *i.e.* Tri-'angles, it gives Rules to determine not only 'the proportion of their ſides, but alſo that of 'their capacity, *Area* or ſuperficies. In the 'next place we learn from it how to find out 'proportional lines, and to augment or diminiſh 'any figure, according to any proportion aſſign'd. 'Here alſo is demonſtrated the moſt uſeful Rule 'of Three; and the Forty-ſeventh of the Firſt 'extended to any figure whatſoever. Laſtly, it 'lays down the moſt facile and moſt certain 'Principles to conduct us in taking any manner 'of Dimenſions.

DEFINITIONS.

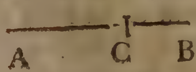


1. RECtilineal figures are similar, when their angles are equal, and the sides, that form those angles, proportional. 'As the triangles ABC, 'DEF, will be similar, if the 'angles A and D, B and E, C 'and F, be equal; and AB has the same proportion to AC as DE to DF, and AB to BC as 'DE to EF.



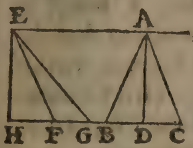
2. Figures are reciprocal, when they may be so compar'd, that the antecedent of one proportion and the consequent of another are both found in the same Figure. 'That is, when 'the Analogy begins and ends in the same figure. As if AB has the same proportion to 'CD, as DE to BF.

3. A line is divided according to the extreme and middle proportion, when the whole line has the same proportion to the greater part, as the greater part to the less. 'As if 'AB has the same proportion



to

'to AC, as AC to CB: the line AB is divided 'according to the extreme and middle propor-'tion.



4. The height of any figure is a perpendicular drawn from its ſummity to its baſe. 'As in the triangles ABC, EFG, 'the perpendiculars EH and 'AD, whether they fall with-'in or without the triangles, are their heights. 'Hence it follows, that all triangles and paral-'lelograms, that have equal heights, may be 'plac'd within the ſame parallels. For having 'ſet their baſes upon the line HC, if the perpen-'diculars DA and HE be equal, the lines EA 'and HC will be parallels.

5. A Proportion is ſaid to be compounded of many others, when the * quantities of thoſe proportions being multiply'd, make another. 'To underſtand the true intent of 'this definition, it muſt be obſerv'd, that every 'Proportion, at leaſt, every * rational Proportion, takes its De-'nomination from a certain num-'ber, denoting that reſpect or relation that the 'antecedent of the proportion bears to the con-'ſequent. As if two magnitudes were propos'd, 'one of twelve foot in length, and the other of 'ſix, we ſhould call that proportion of 12 to

* *Denominators.*

* *Expreſſible by true numbers.*

'6 the double proportion. In like manner if '4 and 12 were propos'd, we ſhould give that 'the name of ſubtriple proportion, $\frac{1}{3}$ being its 'Denominator; importing as much as that the 'proportion of 4 to 12, is the ſame as that of '$\frac{1}{3}$ to unity, or as one to three. This Denom-'nator is call'd the *quantity* of the Proportion. 'Suppoſe therefore three terms were given, 12, '6, and 2; the firſt proportion of 12 to 6 being 'double, its Denominator is two; the ſecond of '6 to 2 being triple, its Denominator is three; 'the proportion therefore of 12, to 2 is ſaid to 'be compounded of that of 12 to 6, and of 6 'to 2, the double and the triple proportion. 'To find therefore the Denominator of the pro-'portion of 12 to 2, multiply three by two, and 'the product ſix will ſhew the proportion of '12 to 2 to be ſextuple, *i. e.* as one to ſix. This 'is that which *Mathematicians* commonly un-'derſtand by compounded Proportion, though 'methinks it might more properly have been 'call'd Proportion multiply'd.

PRO-

PROPOSITION I.

A Theorem.

Parallelograms, and Triangles, of the ſame hight, are in the ſame proportion as their baſes.

Suppoſe the triangles AGC, and DEM, to have the ſame hight, and to be plac'd between the ſame parallels, AD, and GM: I ſay, they will have the ſame proportion as their baſes GC, and EM. Divide the baſe EM into as many equal parts as you pleaſe, and draw lines from the point D to each diviſion, as DF, DH, DL. In like manner divide the line CG into parts equal to thoſe of the line EM, and draw lines from its ſummity A to thoſe diviſions, as AB, AI, &c. All thoſe little triangles being encloſ'd within the ſame Parallels, and having equal baſes, are equal, (*by the* 38 1.)

Demonſtration.

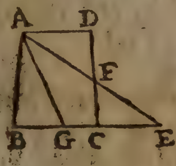
The baſe GC contains ſo many aliquot parts of EM, as there are parts found in it equal to EF; but as many parts equal to EF as are found in the baſe GC, ſo many little triangles are contain'd in the triangle AGC, equal to

thoſe contain'd in DEM; which being equal among themſelves, are the aliquot parts of the triangle DEM. As oft therefore as the baſe GC contains thoſe aliquot parts of EM, ſo oft does the triangle AGC contain the aliquot parts of the triangle DEM; which alſo will happen in every diviſion whatſoever: therefore the triangle AGC has the ſame proportion to the triangle DEM, as the baſe GC to the baſe EM.

Now Parallelograms, deſcrib'd upon the ſame baſes, and encloſ'd between the ſame parallels, are double the triangles, (*by the* 41. 1.) therefore they are in the ſame proportion as the triangles, *i. e.* as their baſes.

The USE.

'This Propoſition is not only ſerviceable in 'demonſtrating thoſe that follow, but alſo of 'great uſe in dividing large Fields, or Plains. 'As for example, ſuppoſe you were to take the 'third part of the *Trapeſium* 'ABCD, having the ſides AD 'and BC parallel; produce the 'line BC to E, ſo that CE 'may be equal to AD; and 'taking BG the third part of 'BE, draw AG: I ſay, the triangle ABG is the third part of the *Trapeſium* 'ABCD.



Demon-

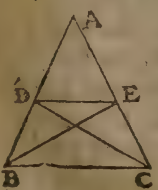
Demonstration.

'The triangles ADF, and FCE, are equiangular, because the lines AD and CE are parallels, and their sides AD and CE are equal: therefore (by the 26. 1.) the triangles are equal, and consequently the triangle ABE is equal to the *trapesium* ABCD. But the triangle ABG is the third part of the triangle ABE, (*by the preceding*,) therefore the triangle ABG is the third part of the *trapesium* ABCD.

PROPOSITION II.

A Theorem.

A line drawn in a triangle parallel to its base divides its sides proportionally; and the line that divides the sides of a triangle proportionally, will be parallel to its base.



IF in the triangle ABC the line DE be parallel to the base BC, the sides AB and AC will be divided proportionally, *i. e.* AD will have the same proportion to DB, as AE to EC. Draw the lines DC and BE. The triangles DBE, and DEC, having the same

base

base DE, and being enclos'd within the same Parallels DE and BC, are equal, (*by the* 37. 1.)

Demonstration.

The triangles ADE and DBE have the same hight E, taking AD and DB for their bases, and therefore may be plac'd within the line AB and another parallel to it drawn through the point E, and consequently have the same proportion as their bases, (*by the* 1.) *i. e.* the triangle ADE has the same proportion to the triangle DBE, or its equal CED, as AD to DB. But the triangle ADE has likewise the same proportion to the triangle CED, as AE to EC; and therefore AD has the same proportion to DB, as AE to EC.

Secondly, suppose AE to have the same proportion to EC, as AD to DB, I say the lines DE and BC will be parallels.

Demonstration.

AD has the same proportion to DB, as the triangle ADE to the triangle DEB, (*by the* 1.) and AE has the same proportion to EC, as the triangle ADE to the triangle CED; and consequently the triangle ADE has the same proportion to the triangle DEB, as the same ADE to the triangle CED. Therefore (*by the* 9. 5.) the triangles BDE, and CED, are equal, and (*by the* 39. 1.) between the same parallels. Therefore the lines DE and BC are parallels.

The

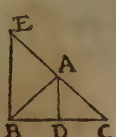
The USE.

'This Propoſition is abſolutely neceſſary for 'the demonſtration of thoſe that follow. It may 'alſo be ſerviceable in taking Dimenſions. As, 'in the following figure, if you were to mea- 'ſure the hight of BE; having erected a ſtaff or 'pole DA, there will be the ſame proportion of 'CD to DA, as of BC to BE.

PROPOSITION III.

A THEOREM.

A line that divides an angle of a triangle into two equal parts, divides its baſe into two parts, which have the ſame proportion as the ſides. And if it divide the baſe into two parts proportional to the ſides, it will divide the angle into two equal parts.



IF the line AD divide the angle BAC into two equal parts, AB will have the ſame proportion to AC as BD to DC. Produce the ſide CA, and take AE equal to AB; then draw the line EB.

Demonſtration.

The external angle CAB is equal to the two internal angles AEB, and ABE: which being equal, (*by the* 5. 1.) becauſe the ſides AE and AB

AB are equal; the angle BAD, which is the half of BAC, will be equal to one of them, ſuppoſe ABE; therefore *(by the* 27. 1.) the lines AD and EB are parallel, and (*by the* 2.) there is the ſame proportion of EA or AB to AC, as of BD to DC.

Secondly, if AB has the ſame proportion to AC as BD to DC, the angle BAC will be divided into two equal parts.

Demonſtration.

AB or EA has the ſame proportion to AC, as BD to DC: therefore EB and AD are parallel; and *(by the* 29. 1.) the alternate angles EBA and BAD, the internal BEA, and the external DAC, will be equal: and the angles BEA and EBA being equal, the angles BAD and DAC will be alſo equal. Therefore the angle BAC will be divided into two equal parts.

The U S E.

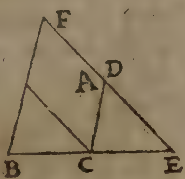
'We make uſe of this Propoſition chiefly to 'find the proportion of the ſides of a triangle.

PRO-

PROPOSITION IV.

A Theorem.

The ſides of equiangular triangles are proportional.



IF the triangles ABC, and DCE, be equiangular, *i. e.* if the angles ABC and DCE, BAC and CDE be equal, AB will have the ſame proportion to BC as DC to CE, and alſo AC will have the ſame proportion to CB as DE to EC. Join the triangles after ſuch a manner, that the baſes BC and CE may be upon the ſame line, and produce the ſides BA and ED till they meet in F; ſince the angles ACB and DEC are equal, the lines AC and FE are parallels, [*by the* 28. 1.] and by the ſame reaſon CD and BF are parallels, and therefore AFDC a Parallelogram.

Demonſtration.

In the triangle BFE, AC is parallel to FE, therefore [*by the* 2.] AB has the ſame proportion to AF, or, which is equal to it, CD, as BC to CE: and alternatively, AB has the ſame proportion to BC, as CD to CE. In like manner in the ſame triangle, CD being parallel to BF,

BF, FD or AC has the ſame proportion to DE, as *BC* to *CE*, [*by the* 2.] and alternatively, *AC* has the ſame proportion to *BC*, as *DE* to *CE*.

The USE.

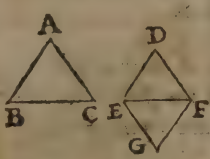
'This Propoſition is of ſo general uſe, that it 'may paſs for a moſt univerſal principle in ta-'king all manner of Dimenſions. For in the firſt 'place, all the methods of meaſuring inacceſſible 'lines, by deſcribing a ſmall triangle ſimilar to 'that which is form'd upon the ground, is 'founded upon it; as alſo the greateſt part of 'thoſe *Mathematical Inſtruments*, upon which 'are deſcrib'd triangles, ſimilar to thoſe of which 'we deſire to take the dimenſions, as the *Geo-'metrical ſquare*, the *Pantometer*, the *Arbaleſt* 'or *Croſs-ſtaff*, &c. Nor could we know how to 'raiſe the Plane of any place, but by the help 'thereof. So that in fine, to unfold all the uſes 'of this Propoſition, it would be neceſſary to 'tranſcribe the whole firſt Book of *Practical Ge-'ometry*.

PRO-

PROPOSITION V.

A THEOREM.

Triangles, which have proportional ſides, are equiangular.



IF the triangles ABC and DEF have proportional ſides, *i. e.* if AB has the ſame proportion to BC as DE to EF, and alſo AB the ſame proportion to AC as DE to DF; the angles ABC and DEF, A and D, C and F, will be equal. Make the angle FEG equal to the angle B, and EFG equal to the angle C.

Demonſtration.

The triangles ABC and EFG have two angles equal, therefore (*by Coroll.* 2. *of the* 32. 1.) they are equiangular; and (*by the* 4.) AB has the ſame proportion to BC as GE to EF. Now 'tis ſuppos'd, that DE has the ſame proportion to EF as AB to BC, therefore DE has the ſame proportion to EF as EG to EF; and conſequently (*by the* 9. 5.) DE and EG are equal. After the ſame manner DF may be prov'd equal to FG, and conſequently (*by the* 8. 1) the triangles DEF and GEF are equiangular. But the angle

angle *GEF* was made equal to *B*, therefore the angle *DEF* is equal to the angle *B*, and the angle *DFE* to the angle *C*; and consequently, the triangles *ABC* and *DEF* are equiangular.

PROPOSITION VI.

A Theorem.

Triangles, which have each one of their angles equal to one of the other, and the sides containing that angle proportional, are equiangular.

If the triangles *ABC* and *DEF* (*see fig. preced.*) have the angles *B* and *E* equal, and the side *AB* has the same proportion to *BC* as *DE* to *EF*, the triangles *ABC* and *DEF* will be equiangular. Make the angle *FEG* equal to the angle *B*, and the angle *EFG* equal to the angle *C*.

Demonstration.

The triangles *ABC* and *GEF* are equiangular, (*by Coroll.* 2. *of the* 32. 1.) therefore *AB* has the same proportion to *BC* as *GE* to *EF*, (*by the* 4.) But as *AB* to *BC*, so is *DE* to *EF*; therefore *DE* has the same proportion to *EF* as *GE* to the same *EF*; and therefore (*by the* 9. 5.) *DE* and *EG* are equal; and the triangles *DEF* and *GFE*, having the angles *DEF* and *GEF*, each equal to the angle *B*, and the sides *DE*

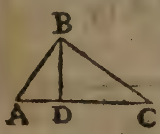
DE and *EG* equal, with the ſide EF common to both, will be equal in all reſpects, *(by the* 4. 1.*)* therefore they will be equiangular; and the the triangle *EGF* being equiangular to *ABC*, the triangles *ABC* and *DEF* are equiangular.

The Seventh Propoſition is of no uſe.

PROPOSITION VIII.

A THEOREM.

A perpendicular, drawn from the right angle of a rectangular triangle to the oppoſite ſide, divides the triangle into two others ſimilar to it.



IF the perpendicular *BD* be drawn from the right angle *ABC* to the oppoſite ſide *AC*, it will divide the rectangular triangle *ABC* into two triangles *ADB* and *BDC*, which will be ſimilar, or equiangular to the triangle *ABC*.

Demonſtration.

The triangles *ABC* and *ADB* have the ſame angle *A*, and the angles *ABC* and *ADB* right angles, therefore they are equiangular, [*by Coroll.* 2. *of the* 32. 1.] In like manner the triangles *BDC* and *ABC* have the angle C common to both, and the angles *ABC* and *BDC* right angles; therefore they alſo are equiangular. There-

Therefore the triangles ABC, BDC, and ADB, are ſimilar triangles.

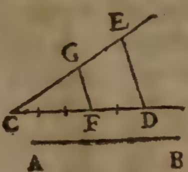
The USE.

'By the help of this Propoſition inacceſſible 'diſtances may be meaſured by a Carpenter's 'Rule. As for example, if I were to meaſure 'the diſtance *D C*; having drawn the perpen-'dicular *B D*, and plac'd my ſquare upon the 'point B in ſuch a manner, that by one of its 'ſides BC I could obſerve the point C, and by 'the other the point A: 'tis evident, there 'would be the ſame proportion of AD to DB, 'as of DB to DC. Therefore multiplying DB 'by its ſelf, and dividing the product by AD, 'the Quotient would be DC.

PROPOSITION IX.

A PROBLEM.

Of a line given, to cut off what part you pleaſe.



LEt the line propos'd be AB, from which you de-ſire to take away three fifth parts. Make an Angle ECD, and upon one of its ſides CD take five equal parts, three of which ſhall be contain'd in CF:

CF: then taking CE equal to AB, draw the line DE, and another parallel to that FG: the line CG will contain three fifth parts of CE or AB.

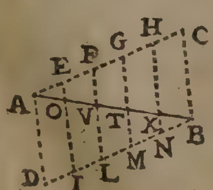
Demonſtration.

In the triangle ECD, FG being parallel to the baſe DE, CF will have the ſame proportion to FD as CG to GE, [*by the* 2.] and compounding them, [*by the* 18. 5.] CE will have the ſame proportion to CG as CD to CF; and (*by the Coroll.* of the 16. 5.) CG will have the ſame proportion to CE as CF to CD. But CF contains three fifths of CD, therefore CG will contain three fifths of CE or AB.

PROPOSITION. X.

A Problem.

To divide a line after the ſame manner as another line given is divided.



If you would divide the line AB after the ſame manner as AC is divided, make with the two lines the angle CAB of what magnitude you pleaſe; then draw the line BC, and parallel to

it the lines EO, FV, and the reſt. The line AB will be divided after the ſame manner that AC is.

Demonſtration.

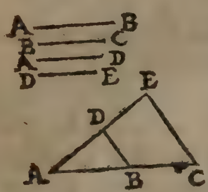
Since in the triangle BAC, HX is drawn parallel to the baſe BC, it will divide the ſides AB and AC proportionally, (*by the* 2.) and the ſame may be ſaid of all the reſt.

To do this more eaſily, you may draw the line BD parallel to AC; and transferring the diviſions of AC to BD, draw lines from one to the other.

PROPOSITION XI.

A PROBLEM.

Two lines being given, to find a third proportional.



IF you would find a third proportional to the lines AB and BC, *i. e.* that there may be the ſame proportion of AB to BC, as of BC to the line ſought; make at pleaſure the angle EAC, and upon one of its ſides take the lines AB and BC, one immediately after the other; and upon the other ſide take AD equal to BC: then draw the line BD, and parallel to it the line CE; and the line DE will be that which you ſeek.

De-

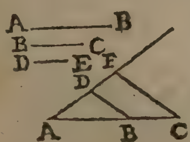
Demonſtration.

In the triangle EAC the line DB is parallel to the baſe CE, therefore [*by the* 2.] there is the ſame proportion of AB to BC, as of AD or BC to DE.

PROPOSITION: XII.

A PROBLEM.

Three lines being given, to find a fourth proportional.



LET the three lines propos'd, to which you are to find a fourth proportional, be AB, BC, and DE. Make at pleaſure the angle FAC, and take upon AC the lines AB and BC, and upon AF the line AD equal to DE; then draw the line DB, and parallel to it FC: I ſay, that DF is the line ſought, *i. e.* there is the ſame proportion of AB to BC, as of DE, or AD to DF, [*by the* 2.]

Demonſtration.

In the triangle FAC the line DB is parallel to the baſe FC: there is therefore the ſame proportion of AB to BC, as of AD to DF, [*by the* 2.]

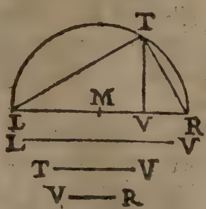
The USE.

'The uſe of the *Compaſs* of *Proportion* is 'grounded upon theſe four Propoſitions; for 'that Inſtrument teaching us to divide a line as 'we pleaſe; to make uſe of the *Rule of Three*, 'without the help of *Arithmetick*; to extract 'the Square, and Cubick roots; to double the 'Cube; to meaſure all ſorts of Triangles; to 'find the capacity of Superficies's, and the ſoli-'dity of bodies; and to augment or diminiſh 'any figure, according to what proportion we 'deſire; all theſe operations are demonſtrated by 'the preceding Propoſitions.

PROPOSITION XIII.

A Problem.

Two lines being given, to aſſign a middle proportional.



IF you deſire a middle proportional between the lines LV, and VR; having plac'd them ſo, that they make but one right line LR, divide that line into two equal parts in M; and having

ing deſcrib'd a ſemicircle LTR from the center M, draw the perpendicular VT, which will be a midle proportional between LV and VR. Draw the lines LT, and TR.

Demonſtration.

The angle LTR, being deſcrib'd in a ſemicircle, is a right angle, (*by the* 31. 3.) and (*by the* 8.) the triangles LVT and TVR are ſimilar; therefore there is the ſame proportion of LV to VT in the triangle LVT, as of VT to VR in the triangle TVR, (*by the* 4.) therefore VT is a middle proportional between LV and VR.

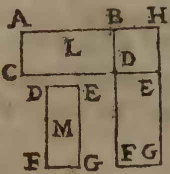
The USE.

'By this Propoſition any rectangular Parallelogram may be reduc'd to a Square. For example, in the Rectangle contain'd under LV and VR, the ſquare of VT is equal to the Rectangle under LV and VR; as I all hereafter demonſtrate.

PROPOSITION XIV.

A Theorem.

Equal equiangular Parallelograms have their ſides reciprocal; and equiangular Parallelograms, that have their ſides reciprocal, are equal.



IF the Parallelograms L and M be equiangular and equal, their ſides will be reciprocal, *i.e.* *CD* will have the ſame proportion to *DE* as *FD* to *DB*. Since they have equal angles, they may be ſo joyn'd together, that the ſides *CD* and *DE*, *FD* and *DB*, will concur in two right lines, *(by the Coroll. of the* 15. 1.) producing therefore the ſides *AB* and *GE*, you compleat the Parallelogram *BDEH.*

Demonſtration.

The Parallelograms *L* and *M* being equal, will have the ſame proportion to the Parallelogram *BDEH*. But the proportion of *L* to *BDEH*, is as the baſe *CD* to the baſe *DE*; and that of *M*, or *DFGE*, to *BDEH*, is as the baſe *FD* to the baſe *DB*, *(by the* 1.*)* Therefore *CD* has the ſame proportion to *DE*, as *FD* to *DB*.

Se-

Secondly, if the Parallelograms L and M be equiangular, and have their ſides reciprocal, they will be equal.

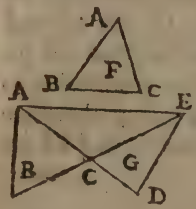
Demonſtration.

The ſides of the Parallelograms are ſuppos'd to be reciprocal, *i. e.* that there is the ſame proportion of CD to DE, as of FD to DB: but as the baſe CD is to DE, ſo is the Parallelogram L to the Parallelogram BDEH, (*by the* 1.) and as FD to DB, ſo is the Parallelogram M to BDEH; therefore L has the ſame proportion to BDEH, as M to the ſame BDEH; therefore (*by the* 9. 5.) the Parallelograms L and M are equal.

PROPOSITION XV.

A Theorem.

Equal triangles, that have one angle equal each to the other, have the sides that form that angle, reciprocal; and if those sides be reciprocal, they will be equal.



If the triangles F and G, being equal, have the angles ACB and DCE equal, their sides that form those angles will be reciprocal, *i. e.* BC will have the same proportion to CE as CD to CA. Place the triangles so, that the sides CD and CA may make one right line; and then because the angles ACB and DCE are suppos'd to be equal, BC and CE will also make one right line, (*by Coroll. of the* 15. 1.) draw the line AE.

Demonstration.

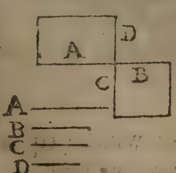
The triangle ABC has the same proportion to the triangle ACE, as the triangle ECD, equal to the former, to the same ACE, (*by the* 7. 5.) But as ABC to ACE, so is the base BC to the base CE, [*by the* 1.] having both the same hight A; and as ECD to ACE, so is the base CD

CD to CA, (*by the ſame*:) therefore BC has the ſame proportion to CE, as CD to CA. But if the ſides be ſuppos'd reciprocal, *i. e.* that BC has the ſame proportion to CE as CD to CA, the triangles ABC and CDE will be equal, becauſe they will both have the ſame proportion to ACE.

PROPOSITION XVI.

A THEOREM.

If four lines be proportional, the rectangle contain'd under the firſt and the fourth, will be equal to the rectangle contain'd under the ſecond and the third. And if the rectangle contain'd under the extremes be equal to that contain'd under the middle terms, the four lines will be proportional.



IF the lines A, B, C, D, be proportional, *i. e.* if as A to B, ſo C to D, the rectangle containd under the firſt A, and the fourth D, will be equal to the rectangle contain'd under B and C.

Demonſtration.

The rectangles have one angle equal each to th' other, becauſe 'tis a right angle in both; their ſides alſo are reciprocal: therefore they are equal, [*by the* 14.]

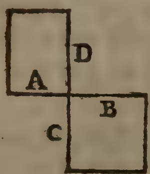
In

In like manner, if they are equal, their ſides will be reciprocal, *i. e.* A will have the ſame proportion to B as C to D.

PROPOSITION XVII.

A THEOREM.

If three lines be proportional, the rectangle contain'd under the firſt and the third, will be equal to the ſquare of the middle term. And if the ſquare of the middle term be equal to the rectangle under the extremes, the three lines are proportional.



IF the three lines A, B, D, be proportional, the rectangle contain'd under A and D will be equal to the ſquare of B. Take C equal to B, and there will be the ſame proportion of A to B as of C to D; therefore the four lines are proportional.

Demonſtration.

The rectangle under A and D will be equal to that under B and C, (*by the preceding*) but the laſt rectangle is a ſquare, the lines B and C being equal: therefore the rectangle contain'd under A and D is equal to the ſquare of B.

In like manner, if the rectangle under A and D

D be equal to the ſquare of B, A will have the ſame proportion to B as C to D: and B and C being equal, A will have the ſame proportion to B, as B to D.

The USE.

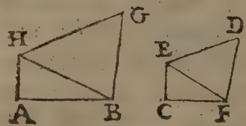
'By theſe four Propoſitions may be demonſtrated that Rule in *Arithmetick*, which is commonly call'd the *Rule of Three*; and conſequently, the Rules of *Fellowſhip*, of *Falſe*, and all thoſe others that depend upon Proportion. For example, Suppoſe Three numbers given, A 8, B 6, and C 4, and it be requir'd to find a fourth proportional number; which taking as found, I will call D. The rectangle then contain'd under A and D, is equal to that under B and C. But I may have this latter rectangle by multiplying B by C, *i. e.* ſix by four, the product will be twenty four; therefore the rectangle contain'd under A and D is alſo twenty four; and therefore dividing that number by A, which is 8, the Quotient three will be the number ſought.

PRO-

PROPOSITION XVIII.

A PROBLEM.

To describe a Polygon similar to another upon a line given.



LEt AB be the line assign'd, upon which you are requir'd to describe a Polygon similar to the Polygon CFDE; and having divided the Polygon CFDE into triangles, upon the line AB make a triangle ABH similar to the triangle CFE, *i. e.* make the angle ABH equal to the angle CFE, and BAH equal to FCE, for then the triangles ABH and CFE will be equiangular, [*by Corol.2. of the* 32. 1.] Make also upon the line BH a triangle equiangular to FDE.

Demonstration.

Since the triangles, which are part of the Polygons, are equiangular, the two Polygons are equiangular. Further, since the triangles ABH and CFE are equiangular, AB will have the same proportion to BH as CF to FE, [*by the* 4.] In like manner, the triangles HBG and EFD being equiangular, BH will have the same proportion to BG as FE to FD: and by equality, (accord-

[*according to defin.* 18. 5.] AB will have the ſame proportion to BG, as CF to FD. And the ſame may be ſaid of all the other ſides. Therefore (*by defin.* 1.) the Polygons are ſimilar.

The U S E.

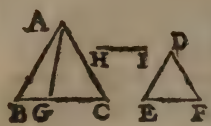
‘ Upon this propoſition is grounded the great-‘ eſt part of *Practical Geometry*, that relates to ‘ the raiſing the plane of any place, as of a ‘ building, field, foreſt, or a whole Country. For ‘ having divided a line into equal parts, to an-‘ ſwer the feet or yards contain'd in the plane, ‘ you may deſcribe a figure ſimilar to, but leſs ‘ than the Original, in which you may ſee the ‘ proportion of all its lines. And having by ex-‘ perience found it much more eaſie to travel ‘ upon paper, than to take a tedious journey ei-‘ ther by land or water, this propoſition will ‘ likewiſe afford us aſſiſtance in this reſpect, in-‘ forming us in almoſt all the parts of *Geodeſia*, ‘ and *Chorography*; and giving Inſtructions how ‘ to compoſe *Geographical Charts*, and *Maps*; ‘ which are nothing elſe but methods of redu-‘ cing great figures to ſmall. Further, the uſe of ‘ this Propoſition extends it ſelf to almoſt all ‘ thoſe Arts, that require the deſign and model of ‘ their works before-hand.

PRO-

PROPOSITION XIX.

A THEOREM.

Similar triangles are in the duplicate proportion of their homologous sides.



IF the triangles ABC and DEF be similar or equiangular, they will be in the duplicate proportion of their homologous sides BC, EF, *i. e.* the proportion of the triangle ABC to the triangle DEF will be the duplicate of the proportion of BC to EF; so that finding a third proportional HI to the lines BC and EF, and making BC to have the same proportion to EF as EF to HI, the triangle ABC will have the same proportion to DEF as the line BC to the line HI; which is to have to it a duplicate proportion, [*by defin.* 11.5.] Take BG equal to HI, and draw the line AG.

Demonstration.

The angles B and E of the triangles ABG and DEF are equal; and besides, since the triangles ABC and DEF are similar, AB will have the same proportion to DE as BC to EF, [*by the fourth.*] But as BC to EF, so EF to HI or BG; therefore as AB to DE, so EF to BG; and

and consequently, the sides of the triangles ABG and DEF being reciprocal, the triangles will be equal, [*by the* 15.] And [*by the* 1.] the triangle ABC has the same proportion to the triangle ABG, as BC to BG or HI: therefore the triangle ABC has the same proportion to the triangle DEF, as BC to HI.

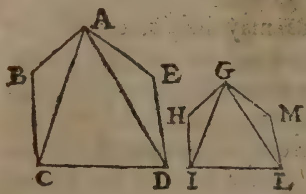
The USE.

'These Propositions may help to correct the 'error of those, who are apt to imagine similar 'figures to have the same proportion as their 'sides. For if two squares, two pentagons, two 'hexagons, or two circles, be propos'd, and the 'side of the first be double that of the second, 'the first figure will be quadruple the second: if 'the side of the first be triple that of the se'cond, the first figure will be nine times greater 'than the second. Therefore to make a square 'triple to another, you must seek a middle pro'portional between one and three, and you'll 'find for the side of your triple figure almost $1\frac{3}{4}$.

PROPOSITION XX.

A Theorem.

Similar Polygons may be divided into an equal number of triangles, and are in the duplicate proportion of their homologous ſides.



If the Polygons ABCDE and GHILM be ſimilar, they may be divided into an equal number of ſimilar triangles, which will be the ſimilar parts of their wholes. Draw the lines AC, AD, GI, GL.

Demonſtration.

Since the Polygons are ſimilar, their angles B and H will be equal; and AB will have the ſame proportion to BC as GH to HI, (*by defin.* 1.) therefore the triangles ABC and GHI are ſimilar, (*by the* 6.) and (*by the* 4) BC has the ſame proportion to CA as HI to GI. Further, becauſe CD has the ſame proportion to BC as IL to IH, and BC the ſame to CA as HI to IG; by equality, CD will have the ſame proportion to CA as IL to GI. Now the angles BCD

BCD and HIL being equal, if the angles ACB and GIH, which are equal, be taken from them, the angles *ACD* and GIL will remain equal. Therefore (*by the* 6.) the triangles ACD and GIL will be similar. In like manner, 'tis easie to run over all the triangles of the Polygons, and to prove them similar.

I add further, that the triangles are in the same proportion as the Polygons.

Demonstration.

Since all the triangles are similar, their sides will be proportional, (*by the* 4) but each triangle to its similar is the duplicate proportion of the homologous sides, (*by the* 19.) therefore every triangle of one Polygon to every triangle of the other is in the duplicate proportion of its sides; which being the same, the duplicate proportion must be the same; and there will be the same proportion of each triangle to its similar, as of all the triangles of one Polygon to all the triangles of the other Polygon, (*by the* 12.5.) *i. e.* of one Polygon to the other.

Coroll. 1. Similar Polygons are in the duplicate proportion of their sides.

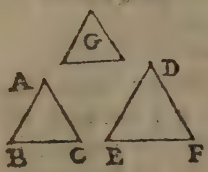
Coroll. 2. If three lines be in continual proportion, a Polygon describ'd upon the first will have the same proportion to a Polygon describ'd upon the second, as the first line to the third, *i. e.* it will be in the duplicate proportion of that of the first line to the second.

T PRO-

PROPOSITION XXI.

A THEOREM.

Polygons, that are ſimilar to another Polygon, are ſo alſo amongſt themſelves.



IF two Polygons be ſimilar to a third, they will be ſo alſo betwixt themſelves. For they may each be divided into as many ſimilar triangles, as are in the third. But triangles ſimiliar to a third, are alſo ſimilar amongſt themſelves; becauſe angles equal to a third, are equal amongſt themſelves; and the angles of the triangles being equal, thoſe of the Polygons being compounded of them muſt be ſo likewiſe.

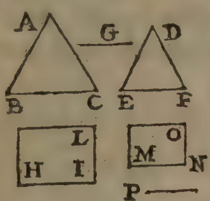
I add, that the ſides of the triangles being proportional, thoſe of the Polygons muſt be ſo alſo, becauſe they are the ſame.

PRO-

PROPOSITION. XXII.

A Theorem.

Similar Polygons describ'd upon four proportional lines, are also proportional. And if the Polygons be proportional, the lines are so too.



IF BC has the same proportion to EF as HI to MN, the Polygon ABC will also have the same proportion to the similar Polygon DEF, as HL to its similar Polygon MO. Seek a third proportional G to the lines BC and EF, and to the lines HI and MO another third proportional P, (*by the* 11) Since BC has the same proportion to EF as HI to MN, and EF to G as MN to P; by equality, BC will have the same proportion to G, as HI to P: and this proportion will be the double of that of BC to EF, or HI to MN.

Demonstration.

The Polygon ABC to the Polygon DEF is in the duplicate proportion of that of BC to EF, (*by the* 20.) that is, as BC to G; and the Polygon HL has the same proportion to MO, as HI to P. Therefore ABC has the same pro-

portion to DEF, as HL to MO.

And if the ſimilar Polygons be proportional, the lines being in the ſubduplicate proportion to them, will be alſo proportional.

The USE.

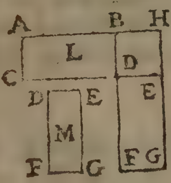
A,	B;	C,	D,
3,	2;	6,	4
9,	4;	36,	16,
E,	F;	G,	H,

This Propoſition may be eaſily apply'd to numbers, if the 'numbers A, B, C, D, be pro'portional, their ſquares E, F, 'G, H, will be ſo alſo; which 'is very ſerviceable in *Arithmetick*, and more 'in *Algebra*.

PROPOSITION XXIII.

A Theorem.

Equiangular Parallelograms are in the proportion compounded of the proportions of their ſides.



If the Parallelograms L and M be equiangular, the proportion of L to M will be compounded of that of AB to DE, and that of BD to DF. Joyn the Parallelograms, ſo that their ſides BD and DF may make but one right line, as alſo CD and DE another; which, the Paral-

rallelograms being equiangular, may be done, [*by the Coroll. of the* 15. 1.] and compleat the parallelogram BDEH.

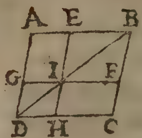
Demonſtration.

The parallelogram L has the ſame proportion to the parallelogram BDEH, as the baſe AB to the baſe BH or DE, [*by the* 1.] and the parallelogram BDEH has the ſame proportion to the parallelogram DFGE, *i. e.* M, as the baſe BD to DF. But the proportion of the parallelogram L to the parallelogram M is compounded of that of L to the parallelogram BDEH, and of that of BDEH to the parallelogram M. Therefore the proportion of L to M is compounded of that of AB to DE, and that of BD to DF. For example, let AB be 8, DE 5, BD 4, DF 7; and make as 4 to 7, ſo 5 to $8\frac{3}{4}$; by which means you will have three numbers, 8, 5, and $8\frac{3}{4}$, 8 to 5 being the proportion of the parallelogram L to BDEH, which is that of AB to DE; and 5 to $8\frac{3}{4}$ that of the parallelogram BDEH to M. Taking away therefore the middle term five, there will remain 8 to $8\frac{3}{4}$ for the proportion compounded of the two.

PROPOSITION XXIV.

A THEOREM.

In all Parallelograms, thoſe through which the diameter paſſes, are ſimiliar to the great one.



SUppoſe the diameter of the Parallelogram AC paſs thro' the Parallelograms EF, GH: I ſay they are ſimilar to the Parallelogram AC.

Demonſtration.

The Parallelograms AC and EF have the ſame angle B: and becauſe in the triangle BCD, IF is parallel to the baſe DC, the triangles BFI and BCD are equiangular. Therefore [*by the fourth*] BC has the ſame proportion to CD as BF to FI, and conſequently the ſides are in the ſame proportion. In like manner IH being parallel to BC; DH will have the ſame proportion to HI as DC to BC; the angles are alſo equal, all the ſides being parallels: therefore [*by defin.* 1.] the parallelograms EF and GH are ſimilar to the parallelogram AC.

The USE.

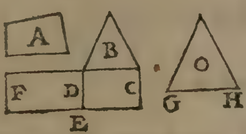
'I have made uſe of this Propoſition to de-'monſtrate the 10th Propoſition of my laſt Book 'of

'of *Perſpectives*, where I have ſhewn a way to
'draw an Image ſimilar to the Original, by a pa
'rallelogram compos'd of four Rules.

PROPOSITION XXV.

A PROBLEM.

To deſcribe a Polygon ſimilar to one Polygon given and equal to another.



If you deſire to deſcribe a Polygon equal to the rectilineal A, and ſimilar to the Polygon *B*, make a parallelogram CE equal to the Polygon B, [*by the* 44. 1.] and upon the line DE make another parallelogram equal to the rectilineal A, [*by the* 45. 1.] Then find a middle proportional GH between CD and DF, [*by the* 13.] Laſtly, make upon GH a Polygon O, ſimilar to B, [*by the* 18.] which will be equal to the rectilineal A.

Demonſtration.

Since CD, GH, and DF, are in a continual proportion, the rectilineal B deſcrib'd upon the firſt, will have the ſame proportion to the rectilineal O deſcrib'd upon the ſecond, as CD to DF, [*by coroll.* 2. *of the* 20.] But as CD to DF, ſo is the parallelogram CE to FE, or B to A,

which are equal to them. Therefore B has the ſame proportion to O as B to A, and conſequently, [*by the* 9. 5.] A and O are equal.

The U S E.

‘ This propoſition teaches how to change one ‘ figure for another, retaining ſtill its equality to ‘ a third; which is very uſeful in *Practical Geo-* ‘ *metry*, for the reducing all figu es to ſquares.

PROPOSITION XXVI.

A THEOREM.

If in one angle of a parallelogram you deſcribe a leſs, ſimilar to the former, the diameter of the greater will fall upon the angle of the leſs.

IF in the angle D of the parallelogram AC you deſcribe a leſſer parallelogram DG, ſimilar to the other, the diameter BG will paſs by the point G. For if it do not paſs by that point, ſuppoſe it then to paſs by the point I, and to make the line BID. Draw the line IE parallel to HD.

Demonſtration.

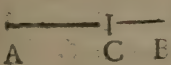
The parallelogram DI would be ſimilar to the

the parallelogram AC, (*by the* 24.) But the parallelogram DG is alſo ſuppos'd ſimilar to it, therefore the parallelograms DI and DG would be ſimilar, which is impoſſible; for ſo HI would have the ſame proportion to IE or GF, as HG to the ſame GF; and (*by the* 9. 5.) the lines HI and HG would be equal.

PROPOSITION XXX.

A PROBLEM.

To divide a line according to the extreme and middle proportion.



LET AB be the line propos'd to be divided according to the extreme and middle proportion, *i. e.* ſo, that AB may have the ſame proportion to AC as AC to CB. Divide the line AB (*by the* 11. 2.) ſo, that the rectangle contain'd under AB and CB may be equal to the ſquare of AC.

Demonſtration.

Since the rectangle under AB and CB is equal to the ſquare of AC, AB will have the ſame proportion to AC as AC to CB, (*by the* 17.)

The USE.

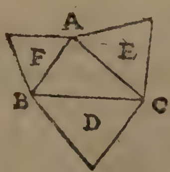
'This propoſition is neceſſary in the Thir-
'teenth Book of *Euclid*, for the finding the ſide
'of

'of five regular bodies. And Friar *Lucas*, of the
'*Holy Sepulcher*, has compos'd a whole Book
'concerning the properties of a Line divided ac-
'cording to the extreme and middle proportion.

PROPOSITION XXXI.

A Theorem.

A Polygon describ'd upon the base of a rectangular triangle, is equal to the two similar Polygons describ'd upon the other sides of the same triangle.



If the angle BAC of the triangle ABC be a right angle, the polygon D, describ'd upon its base BC, will be equal to the two similar polygons F and E describ'd upon the sides AB and AC.

Demonstration.

The polygons D, E, and F, are amongst themselves in the duplicate proportion of their homologous sides BC, AC, and AB, (*by the* 20.) and if squares were describ'd upon the same sides, they also would amongst themselves be in the duplicate proportion of their sides; but [*by the* 47. 1.] the square of BC would be equal to the

the ſquares of AC and AB : therefore the polygon D deſcrib'd upon the baſe BC, will be equal to the ſimilar polygons E, and F, deſcrib'd upon AB and AC.

The USE.

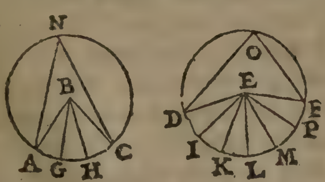
'This propoſition is made uſe of to augment 'or diminiſh all manner of figures, being more 'univerſal than the 47. 1. which yet is exceeding uſeful, in as much as almoſt all *Geometry* is 'grounded upon that principle.

The 32. Propoſition is uſeleſs.

PROPOSITION XXXIII.

A THEOREM.

In equal circles, the angles as well at the center as circumference, as alſo the ſector, are in the ſame proportion as the arches upon which they ſtand.



If the circles ANC and DOF are equal, the angle ABC will have the ſame proportion to the angle DEF as the arch AC to the arch DF. Suppoſe AG, GH, and HC, to be equal arches, and

and consequently the aliquot parts of AC; and let DF be divided into as many parts, equal to AG, as it contains; and draw the lines EI, EK, and the rest. *Demonstration.*

All the angles, ABG, GBH, HBC, DEI, IEK, and the rest, are equal, (*by the* 27. 3.) so that AG, an aliquot part of the arch AC, will be contain'd in the arch DF, as oft as the angle ABG, an aliquot part of the angle ABC, is contain'd in the angle DEF; therefore the arch AC will have the same proportion to the arch DF, as the angle ABC to the angle DEF. And because N and O are the halves of the angles ABC and DEF, they will be in the same proportion as these: therefore the angle N has the same proportion to the angle O, as the arch AC to the arch DF.

The same holds likewise of the Sectors: for if you draw the lines AG, GH, HC, DI, IK, and the rest, they will be equal, (*by the* 29. 3.) and each little sector will be divided into a triangle, and a segment. But the triangles will be equal, (*by the* 8. 1.) and the little segments will also be equal, (*by the* 24. 3.) therefore the whole little Sectors will be equal; and consequently, as many aliquot parts of the arch AC as are contain'd in the arch DF, so many aliquot parts of the sector ABC will be contain'd in the sector DEF. Therefore the arch has the same proportion to the arch, as the sector to the sector.

THE

THE ELEVENTH BOOK OF THE ELEMENTS OF EUCLID.

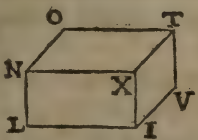
' THis Book eſtabliſhes the firſt principles relating to ſolid bodies; inſomuch that it ' is impoſſible to know any thing certainly concerning the third *ſpecies* of quantity, without ' underſtanding what is herein taught. Upon ' which account the knowledge of it is abſolutely neceſſary to a through inſight into the greateſt part of *Mathematical* Treatiſes. In the firſt ' place, the Doctrin of the Sphere deliver'd by ' *Theodoſius* does ſuppoſe a perfect knowledge ' of the whole. In like manner *Spherical Trigonometry*, the third part of *Practical Geometry*, ' divers propoſitions of *Staticks* and *Geography* ' are built upon the principles of Solids The ' main difficulties in *Gnomonicks*, *Conick Sections*,

'*ctions*, and the Tracts concerning the cutting of
'pretious Stones, arising chiefly from their emi-
'nencies and rais'd parts, not easily represented
'upon paper, and their being contain'd under
'many superficies, are render'd intelligible and
'easie by the previous knowledge of the doctrin
'of Solids.

'I have omitted the seventh, eighth, ninth,
'and tenth Books of the *Elements* of *Euclid*, be-
'ing of little or no use in any part of the *Ma-*
'*thematicks*. And I have oft wondred how they
'obtain'd a place amongst the *Elements*, since 'tis
'evident *Euclid* compil'd them for no other end,
'but to settle the Doctrin of Incommensurables;
'which being little better than a vain curiosity,
'ought not to be receiv'd into the Books which
'treat of the *First Principles* of the Science, but
'to make a particular Treatise by its self. The
'same may be said of the thirteenth Book, and
'those that follow it. And therefore 'tis my o-
'pinion, that almost all parts of the *Mathema-*
'*ticks* may sufficiently be understood by the help
'of these eight Books of the *Elements* of *Eu-*
'*clid*.

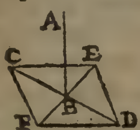
DE-

DEFINITIONS.

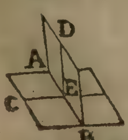


1. A Solid body is a quantity, that hath length, breadth, and depth, or thickneſs. ‘As the figure LT, whoſe length is ‘NX, breadth NO, and thickneſs LN.

2. The extremes or terms of a ſolid body are ſuperficies's.



3. A Line is right, or perpendicular to a plane, when 'tis perpendicular to all the lines, which it meets in the plane. ‘As the ‘line AB will be right to the ‘plane CD, if it be perpendicular to the lines ‘CD and FE, which being drawn upon the ‘plane CD, paſs by the point B, ſo that the ‘angles ABC, ABD, ABE, and ABF, are right ‘angles.

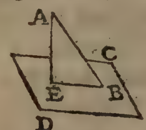


4. One plane is perpendicular to another, when a perpendicular line drawn upon one of them to the common ſection, is alſo perpendicular to the other.

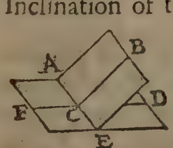
‘We call the line that is com-

‘mon

‘mon to both the planes the common ſection of ‘the planes : As the line AB, which is as well ‘in the plane AC, as in the other AD. If there- ‘fore the line DE, drawn on the plane AD, ‘perpendicular to AB, be alſo perpendicular to ‘plane AC, the plane AD will be right to the ‘plane AC.



5. If the line AB be not perpendicular to the plane CD, and from the point A a perpendicular be drawn to it AE, and alſo the line BE; the angle ABE is the Inclination of the line AB to the plane CD.



6. The Inclination of one plane to another, is the acute angle form'd by the two perpendiculars drawn upon each plane to their common ſection. ‘As the Inclination of the plane AB to the plane ‘AD, is nothing elſe but the angle BCD, ‘form'd by the lines BC and CD, drawn upon ‘the two planes, perpendicular to their common ‘ſection AE.

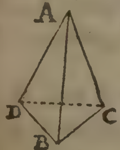
7. Planes are inclin'd after the ſame manner, if their angles of Inclination be equal.

8. Planes are parallel, if being continu'd as far as you pleaſe, they ſtill retain the ſame diſtance one from the other.

9. Solid figures are ſimilar, which are contain'd within, or terminated by, an equal number

ber of ſimilar planes; as two Cubes. "This "definition does not agree to thoſe figures, "whoſe ſuperficies's are crooked; as the Sphere "the Cylinder, and the Cone.

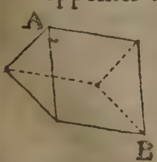
10. Equal and ſimilar ſolid figures are contain'd within, or terminated by, an equal number of equal and ſimilar planes. "Inſomuch, "that if they were ſuppos'd to penetrate each "other, neither of them would exceed, having "their ſides and angles equal.



11. A ſolid angle is the concourſe,* or inclination, of divers lines, in different planes. "As the 'concourſe of the lines AB, AC, 'and AD, which are in different 'planes.

12. A Pyramid is a ſolid figure, terminated by triangles, whoſe baſes are in the ſame plane. "As the figure ABCD.

13. A Parallelpipedon is a ſolid figure contain'd within ſix quadilateral planes, of which the oppoſites are parallel.



14. A Priſm is a ſolid figure, having two parallel planes ſimilar and equal, and the others Parallelograms. "As the "figure AB. Its oppoſite planes "may be Polygons.

15. A Sphere is a ſolid figure, terminated by one only ſuperficies: from which divers

U lines

lines being drawn to a point in the middle of the figure, they will be all equal. ‘ Some de-
‘ fine a Sphere by the motion of a ſemicircle,
‘ turn'd about upon its diameter, which remains
‘ immoveable.

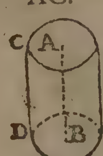
16. The Axis of a Sphere is that immoveable line about which the ſemicircle is turn'd.

17. The Center of the Sphere is the ſame with that of the ſemicircle, by whoſe motion it is made.

18. The Diameter of a Sphere, is any line whatſoever paſſing through its center, and terminated at the ſuperficies.

19. If a line, immoveable at one of its points, taken above the plane of a circle, be mov'd about the circumference, it will deſcribe a Cone. ‘ As if the line AB, being fix'd at the point
‘ A, be mov'd about the circumfe-
‘ rence BED, it will deſcribe the Cone ABED.
‘ The point A will be its ſummity or vertex,
‘ and the circle BED its baſe.

20. The Axis of a Cone, is the line drawn from the vertex to the center of the baſe. “ As
“ AC.



21. If a line be mov'd about the circumference of two parallel circles, ſo that it remains always parallel to a line drawn from the center of one of the circles to that of the other,

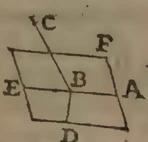
her, *i. e.* the Axis, it will deſcribe a Cylinder

22. Cones are ſaid to be right, when the Axis is perpendicular to the plane of the baſe. Alſo right cones are ſimilar, when their axis's and the diameters of their baſes are in the ſame proportion. But inclin'd cones are not ſimilar, unleſs they have a third condition; that their axis's be equally inclin'd to the planes of their baſes.

PROPOSITION I.

A THEOREM.

A right line cannot have one of its parts upon a plane, and the other above or below it.



IF the line AB be upon the plane AD, it will not, being continu'd, either riſe above or fall below it, but all its parts will lie upon the ſame. For if it be poſſible that BC can be a part of AB continued, draw upon the ſame plane AD the line BD perpendicular to AB, and alſo BE perpendicular to ED upon the ſame.

Demonſtration.

The angles ABD and DBE are two right angles; therefore (*by the* 14 1) AB and BE make but one right line, and conſequently BC is

is no part of the line AB continued: otherwiſe two right lines CB and EB would have the ſame part AB in common, which is repugnant to the 13. Axiom of the firſt Book.

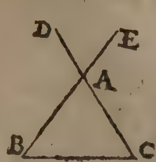
The USE.

'Upon this Propoſition is built a principle in '*Gnomonicks*, by which we prove, that the *ſha-* '*dow* of the *Style* cannot fall out of the plane of 'a great circle, in which is the Sun. For the 'extremity of the *Style* being taken for the cen- 'tre of the heavens, and conſequently of all the 'greater circles, and the *ſhadow* being always in 'a right line of a Ray drawn from the Sun to 'the opacous body, and this ray being in the 'plane of this great circle, the ſhadow muſt be 'ſo likewiſe.

PROPOSITION II.

A Theorem.

Lines that cut each other, are in the ſame plane, as are alſo the parts of a triangle.



IF the two lines BE and CD cut each other at the point A, and a triangle be form'd by drawing the baſe BC; I ſay, all the parts of the triangle ABC are

are in the ſame plane, and alſo the lines BE and CD.

Demonſtration.

It cannot be ſaid that any part of the triangle ABC is in a plane, and another part of the ſame triangle not in the ſame, but it muſt be alſo affirm'd, that one part of a right line is in a plane, and another part of the ſame line is not in the ſame plane; which is contrary to *Prop.* 1. And becauſe the ſides of the triangle muſt be in the ſame plane in which is the triangle, the lines BE and CD will be alſo in the ſame plane.

The USE.

'This Propoſition ſufficiently determines a 'plane, by the concourſe of two right lines, or by 'a triangle. I have alſo made uſe of it in *Op-*'*ticks*, to prove that objective parallel lines, 'which meet upon a Table, ought to be repre-'ſented by lines that concur in a point.

PROPOSITION III.

A THEOREM.

The common ſection of two planes is one right line.

IF the planes AB and CD cut each other, their common ſection EF will be one right line. For if not, take two points common to both planes, as E and F; and draw a right line from the point E to the point F upon the plane AB, which ſuppoſe to be EHF. Draw likewiſe upon the plane CD a right line from the ſame point E to F; and if it be not the ſame with the former, ſuppoſe it to be EGF.

Demonſtration.

Theſe lines drawn upon two planes are two different lines, and encloſe ſpace; which is contrary to the 12. *Axiom of the* 1. Therefore they will make but one right line, which being in both the planes will be their common ſection.

The USE.

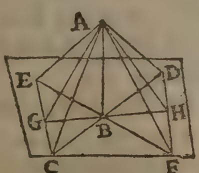
'This is a fundamental propoſition, ſuppos'd in 'divers parts of the *Mathematicks*, though it 'be not always quoted. Particularly, it is ta'ken for granted in *Gnomonicks*, when the hour-lines

lines are repreſented upon *Dials*, by marking ' only the common ſection of their plane, and ' that of the wall.

PROPOSITION. IV.

A Theorem.

If a line be perpendicular to two others that cut each other, it will be alſo perpendicular to the plane of the ſame lines.



IF the line AB be perpendicular to the lines CD and EF, which cut each other at the point B, ſo that the angles ABC, ABD, ABE, and ABF, be right angles, (which cannot conveniently be repreſented upon a plate,) it will be alſo perpendicular to the plane of the lines CD and EF, *i. e.* to all the lines that ſhall be drawn upon the ſame plane through the point B; as, for example, the line GBH. Let the lines BC, BD, BE, and BF, be equal, and draw the lines EC, DF, AC, AD, AE, AF, AG, and AH.

Demonſtration.

The four triangles ABC, ABD, ABE, and ABF, have each a right angle at the point B; and the ſides BC, BD, BE, and BF equal, with the

the ſide AB common to all. Therefore their baſes AC, AD, AE, and AF, are equal, (*by the* 4. 1.)

2. The triangles EBC and DBF will be in all reſpects equal, having their ſides BC, BD, BE, and BF, equal; and the angles CBE and DBF, being oppos'd at the top, equal: therefore the angles BCE, BDF, BEC, and BFD, will be equal, (*by the* 4. 1.) and alſo the baſes EC and DF.

3. The triangles GBC, and DBH, having the oppoſite angles CBG, and DBH, equal; as alſo the angles BDH, and BCG; and the ſides BC and BD; the ſides BG and BH, CG and DH, will be alſo equal, (*by the* 26. 1.)

4. The triangles ACE and AFD, having the ſides AC, AD, AE, and AF, equal, and the baſes EC and DF alſo equal; the angles ADF and ACE will be equal, (*by the* 8. 1.)

5. The triangles ACG and ADH have the ſides AC and AD, CG and DH equal, with the angles ADH and ACG; therefore their baſes AG and AH are equal.

Laſtly, the triangles ABH and ABG have all their ſides equal; therefore (*by the* 8. 1.) the angles ABG and ABH will be equal, and the line AB perpendicular to GH. Accordingly the line AB will be perpendicular to any line drawn through the point B upon the plane of the lines CD and EF, which I call being perpendicular to their plane.

The

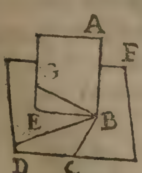
The USE.

'This Propoſition occurs very oft in the firſt 'Book of *Theodoſius*: for example, to demon- 'ſtrate that the *Axis* of the world is perpendi- 'cular to the plane of the *Equinoctial.* In like 'manner in *Gnomonicks*, 'tis demonſtrated by 'this propoſition, that the *Equinoctial line* in '*Horizontal Dials* is perpendicular to the *Me-* '*ridian.* Nor is it leſs uſeful in other *Mathe-* '*matical* Treatiſes; as thoſe concerning *Aſtro-* '*labes*, and the cutting of precious ſtones.

PROPOSITION V.

A THEOREM.

If a line be perpendicular to three others, which cut each other at the ſame point, they will be all three in the ſame plane.



IF the line AB be perpendicular to three lines BC, BD, and BE, which cut each other at the ſame point B, the lines BC, BD, and BE, are in the ſame plane. Suppoſe the plane AE to be that of the lines AB and BE, and CF that of the lines BC and BD. If BE be the common ſection of both the planes, it will be in the plane of

of the lines BC and BD, as was asserted: but if BE be not, let BG be their common section.

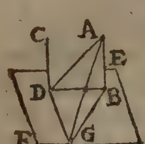
Demonstration.

AB is perpendicular to the lines BC and BD, therefore it is perpendicular to their plane CF, (*by the* 4.) and (*by Defin.* 3.) AB will be perpendicular to BG. But it is also suppos'd perpendicular to BE; therefore the Angles ABE and ABG are right angles, and consequently equal, though one be part of the other. Therefore the two planes can have no other common section but BE, BE is therefore in the plane CF.

PROPOSITION. VI.

A Theorem.

Lines that are perpendicular to the same plane, are parallel.



IF the lines AB and CD be perpendicular to the same plane EF, they will be parallel. 'Tis evident, that the internal angles ABD and BDC are right angles; but that is not enough; it remains to be prov'd, that AB and CD are in the same plane. Draw DG perpendicular to BD, and equal to AB; draw also the lines BG, AG, and AD.

De-

Demonſtration.

The triangles ABD and BDG have the ſides AB and DG equal, and BD common to both: and the angles ABD and BDG are right angles, therefore their baſes AD and BG are equal, (*by the* 4. 1) Further, the triangles ABG and ADG have all their ſides equal: therefore the angles ABG and ADG are equal; and ABG being a right angle, becauſe AB is perpendicular to the plane, ADG is alſo a right angle. Therefore the line DG is perpendicular to three lines CD, DA, and DB, which conſequently are in the ſame plane, (*by the* 5.) but the line AB is in the plane of the lines AD and DB, (*by the* 2.) therefore AB and CD are in the ſame plane.

Coroll. Two parallel lines are in the ſame plane.

The USE.

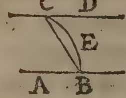
‘ By this propoſition we demonſtrate, that the ‘ hour-lines, in all planes that are parallel to the ‘ *Axis* of the World, as the Polars, Meridional, ‘ and others, are parallel among themſelves.

PRO-

PROPOSITION VII.

A Theorem.

A line drawn from one Parallel to another, is in the same plane with them.



THE line CB, being drawn from the point B of the line AB to the point C of its parallel CD, is (I ſay) in the plane of the lines AB and CD.

Demonſtration.

The parallels AB and CD are in the ſame plane: in which if you draw a right line from the point B to the point C, it will be the ſame with CB; otherwiſe two right lines would encloſe ſpace, contrary to 12. *Axiom of the* 1.

PROPOSITION VIII.

A Theorem.

If one of two parallel lines be perpendicular to a plane, the other will be ſo alſo.

IF of the two parallel lines AB and CD, [*ſee fig. prop.* 6.] the one AB be perpendicular to the plane

plane EF, the other CD will be ſo alſo. Draw the line DB: ſince the angle ABD is a right angle, and AB and CD are ſuppos'd to be parallels, the angle CDB will be a right angle, (*by the* 29: 1.) therefore if I can prove, that the angle CDG is alſo a right angle, it will follow (*by the* 4.) that CD is perpendicular to the plane EF. Make a right angle BDG, and take DG equal to AB; then draw the lines BG and AG.

Demonſtration.

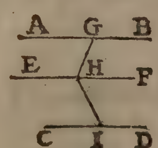
The triangles ABD and BDG have the ſides AB and DG equal, with the ſide BD common to both; and the angles ABD and BDG are right angles: therefore (*by the* 4. 1.) their baſes AD and BG are equal. The triangles ADG and ABG have all their ſides equal, therefore (*by the* 8. 1.) the angles ADG and ABG are equal: But the latter is a right angle, becauſe AB is ſuppos'd to be perpendicular to the plane EF, therefore the angle ADG is a right angle; and the line DG being perpendicular to the lines DB and DA, will be perpendicular to the plane of the lines DB and DA, which is the ſame in which are the parallels AB and CD. Therefore the angle GDC is a right angle, (*by defin.*3.)

PRO-

PROPOSITION IX.

A THEOREM.

Lines, that are parallel to a third, are alſo parallel among themſelves, though not all in the ſame plane.



IF the lines AB and CD are parallel to the line EF, they will be parallel to each other, though all the three lines be not in the ſame plane. Upon the plane of the lines AB and EF draw the line HG perpendicular to AB; which will be alſo perpendicular to EF, [*by the Lemma after the* 26. 1.] In like manner upon the plane of the lines EF and CD draw the line HI perpendicular to EF and CD.

Demonſtration.

The line EH being perpendicular to the lines GH and HI, is ſo alſo to the planes of the lines HG and HI, (*by the* 4.) therefore (*by the* 8.) the lines AG and CI are perpendicular to the plane of the lines HG and HI, and [*by the* 6.] parallel to each other.

The USE.

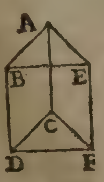
'This propoſition is frequently uſed in *Per-* '*ſpectives*, to determine the repreſentation of 'parallel

‘ parallel lines upon a table; as alſo in the cut-
‘ ting of precious Stones, to prove the ſides of
‘ the Pannels to be parallel among themſelves,
‘ becauſe they are ſo to a line in a different
‘ plane. In *Gnomonicks* likewiſe we are ſome-
‘ times obliged to prove, that the Vertical cir-
‘ cles ought to be deſcrib'd on Walls by perpen-
‘ dicular lines; becauſe the lines, that are the
‘ common ſections of them and the walls, are
‘ parallel to a line drawn from the *Zenith* to the
‘ *Nadir*.

PROPOSITION X.

A THEOREM.

If two lines, which concur, are parallel to two others concurring, of a different plane, they will make equal angles.



IF the lines AB and CD, AE and CF be parallel, though they be not all four upon the ſame plane, yet the angles BAE and DCF will be equal. Let the lines AB and CD, AE and CF be equal, and draw the lines BE, DF, AC, BD, and EF.

Demonſtration.

The lines AB and CD are ſuppos'd to be both parallel and equal, therefore [*by the* 33. 1.] the

the lines AC and BD are parallel and equal, as alſo AC and EF; and [*by the preceding*] BD and EF are parallel, and equal, and conſequently [*by the* 33. 1.] BE and DF will be alſo parallel and equal. Therefore the triangles BAE and DCF have all their ſides equal: and [*by the* 8. 1.] the angles BAE and DCF will be equal.

Coroll. Many the like Propoſitions might be made, which would not be altogether unuſeful: as for example, if upon a parallel plane the line CD be drawn parallel to the line AB, and the angles BCE and DCF be equal, the lines AE and CF will be parallel.

The USE.

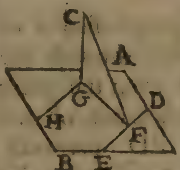
'By this Propoſition we demonſtrate, that 'the angles made by the planes of the hour-'circles with a plane parallel to the *Equator*, 'are equal to the angles made by them with the 'plane of the *Equator*.

PRO-

PROPOSITION XI.

A PROBLEM.

To draw a perpendicular to a plane from a point given out of the plane.



IF you desire to draw a perpendicular from the point C to the plane AB, draw the line EF at pleasure, and CF perpendicular to it, [*by the* 12. 1.] And again [*by the* 11. 1.] upon the plane AB draw FG perpendicular to ED; and CG perpendicular to FG, I say, CG will be perpendicular to the plane AB. Draw GH parallel to FE.

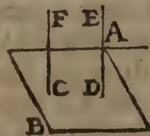
Demonstration.

The line EF being perpendicular to the lines CF and FG, will be perpendicular to the plane CFG, [*by the* 4.] and HG being parallel to EF, will be also perpendicular to the same plane, [*by the* 8.] And because CG is perpendicular to the lines GF and GH, it will be perpendicular to the plane AB, [*by the* 4.]

PROPOSITION XII.

A Problem.

To draw a perpendicular to a plane through a point of the same plane.

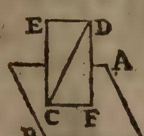


TO draw a perpendicular to the plane AB through the point C, draw from the point E, taken at pleaſure out of the plane, the line ED perpendicular to the ſame plane, [*by the* 11.] Draw alſo [*by the* 31. 1.] CF parallel to DE. CF will be perpendicular to the plane AB, [*by the* 8.]

PROPOSITION XIII.

A Theorem.

Two lines perpendicular to a plane cannot be drawn through the ſame point.



IF the two lines CE and CD, drawn through the ſame point C, were perpendicular to the plane AB, and CF the common ſection of the planes of thoſe lines, with the plane AB; the angles ECF and DCF

DCF would be both right angles, which is impoſſible.

I add, that two perpendiculars DC and DF to the plane AF cannot be drawn from the ſame point D: for having drawn the line CF, there would be two right angles, DCF and DFC, in the ſame triangle, *contrary to the* 32. 1.

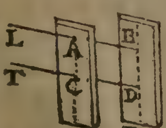
The USE.

'This Propoſition is neceſſary to ſhew, that a 'perpendicular to a plane was ſufficiently de- 'ſcrib'd, in as much as but one ſuch can be 'drawn through the ſame point.

PROPOSITION XIV.

A THEOREM.

Planes, to which the ſame line is perpendicular, are parallel.



IF the line AB be perpendicular to the planes AC and BD they will be parallel, *i. e.* they will in all places be equally diſtant from each other. Draw the line DC parallel to AB, [*by the* 31. 1.] and joyn the lines BD and AC.

Demonſtration.

AB is ſuppos'd to be perpendicular to the

planes AC and BD, therefore the line CD, which is parallel to it, will be alſo perpendicular to them, [*by the* 8] and conſequently the angles B and D, A and C, will be right angles; and [*by the* 28. 1.] the lines AC and BD will be parallels, and the figure ABDC a parallelogram. Therefore the lines AB and CD are equal, [*by the* 34. 1.] *i. e.* the planes in the points A and C, B and D. are equally diſtant. Accordingly the line CD may be drawn through any other point whatſoever; therefore the planes AC and BD are equally diſtant in all places the one from the other.

The USE.

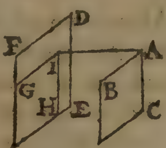
'*Theodoſius* demonſtrates the circles, that have 'the ſame poles, as the *Equator*, and the two '*Tropicks*, to be parallel, becauſe the *Axis* of 'the World is perpendicular to their planes.

PRO-

PROPOSITION XIII.

A THEOREM.

If two lines, meeting at a point, be parallel to two lines of another plane, the planes of those lines will be parallel.



IF the lines AB and AC be parallel to the lines DE and DF, which are in another plane, the planes BC and FE are parallel. Draw AI perpendicular to the plane BC, [*by the* 11.] and GI and IH parallel to FD and DE: they will be also parallel to the lines AB and AC [*by the* 9.)

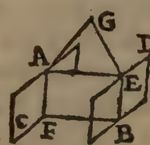
Demonstration.

The lines AB and GI are parallel, and the angle IAB is a right angle, AI being perpendicular to the plane BC: therefore [*by the* 29. 1.] the angle AIG is a right angle, as also the angle AIH. Therefore [*by the* 4.] the line AI is perpendicular to the plane CH; and being also perpendicular to the plane BC, the planes BC and GH, or FE, will be parallel, [*by the* 14.]

PROPOSITION XVI.

A Theorem.

If a plane cut two others which are parallel, their common ſections will together with them be parallel.



If the plane AB cut two other parallel planes, AC and BD; I ſay, their common ſections AF and BE will be parallel. For if not, being continu'd they would at length concur, *e. g.* at the point G.

Demonſtration.

The lines AF and BE are upon the planes AC and BD; and therefore [*by the* 1.] can never be either above, or below it; therefore if they concur at the point G, the planes muſt do ſo likewiſe, and conſequently they would not be parallel, which is contrary to what was ſuppos'd.

The U S E.

'By this Propoſition we demonſtrate, in the 'Treatiſe of *Conick* and *Cylindrick* Sections, 'that if the *Cone* or *Cylinder* be cut by a plane 'parallel to its baſe, the ſections are circular. 'By the ſame we deſcribe *Aſtrolabes*; and 'prove in *Gnomonicks*, that the angles, which

'the

'the hour-circles make with a plane parallel to a 'great circle, are equal to those which they make 'in the circle it self; and again in *Perspectives*, 'that the Images of the objective lines perpendi- 'cular to the table, concur at the point of sight.

PROPOSITION XVII.

A THEOREM.

Two lines are divided proportionally by parallel planes.

IF the lines AB and CD be divided by parallel planes, I say, AE will have the same proportion to EB as CF to FD. Draw the line AD, passing through the plane EF at the point G: Draw also AC, BD, FG, and GE.

Demonstration.

The plane of the triangle ABD cuts the three planes, therefore *(by the* 16.*)* the sections BD and EG are parallel; and *(by the* 2. 6.) AE has the same proportion to EB, as AG to GD. In like manner the plane of the triangle ADC cuts the planes EF and AC, therefore the sections AC and FG are parallel; and FC has the same proportion to FD as AG to GD, *i. e.* as AE to EB.

PROPOSITION XVIII.

A Theorem.

If a line be perpendicular to a plane, all the planes, in which that line is found, are perpendicular to the same plane.

If the line AB be perpendicular to the plane ED, all the planes in which it is found will be perpendicular to the plane ED. Suppoſe AB to be in the plane AE, having for a common ſection with the plane ED the line BE; to which draw a perpendicular FI.

Demonſtration.

The angles ABI and BIF are right angles, therefore the lines AB and FI are parallel; and (*by the* 8.) FI will be perpendicular to the plane ED. Therefore the plane AE will be perpendicular to the plane ED, (*by def.* 4.)

'The ſame may be prov'd of the plane AD.

The USE.

'The firſt Propoſition in *Gnomonicks*, which 'may paſs for a fundamental one, is built upon 'this propoſition; which is alſo frequently 'made uſe of in *Spherical Trigonometry*, in *Per-*

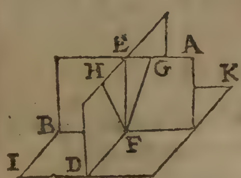
'*ſpectives*,

'*ſpectives*, and generally in all thoſe Treatiſes
'which are oblig'd to conſider divers planes.

PROPOSITION XIX.

A THEOREM.

If two planes cutting each other be perpendicular to another, their common ſection will be perpendicular to the ſame.



IF the planes AB and ED, which cut each other, be perpendicular to the plane IK, their common ſection EF is alſo perpendicular to the plane IK.

Demonſtration.

If EF be not perpendicular to the plane IK, upon the plane AB draw the line GF perpendicular to the common ſection BF: and the plane AB being perpendicular to the plane IK, the line GF will be perpendicular to the ſame plane. Draw likewiſe FH perpendicular to the common ſection DF; it will be alſo perpendicular to the plane IK. We ſhall have therefore two perpendiculars to the ſame plane, drawn through the ſame point F, (*contrary to the*

13.

13. *Propos.*) it must therefore be granted that EF is perpendicular to the plane IK.

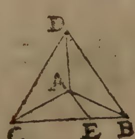
The USE.

'By this Proposition we demonstrate, that the 'circle which passes through the Poles of the 'World and the *Zenith* is the *Meridian*, and 'cuts all the diurnal arches into two equal 'parts; and that the Stars spend as much time 'in their motions from their risings to this cir-'cle, as from the circle to their settings.

PROPOSITION XX.

A THEOREM.

If three plain angles make one solid one, any two of them ought to be greater than the third.



IF the angles BAC, BAD, and CAD, make the solid angle A, and the angle BAC be the greatest angle; the two others, taken together, are greater than BAC. Suppose the angle CAE to be equal to the angle CAD, and the lines AD and AE to be equal; and draw the lines CEB, CD, and BD.

Demonstration.

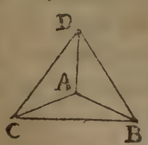
The triangle CAE and CAD have the sides AD and AE equal, and the side AC common to both,

both, and the angles CAD and CAE equal: therefore (*by the* 4. 1.) their bases CD and CE are equal. But the sides CD and DB are greater than the side CB alone, (*by the* 20. 1.) therefore taking away the equal lines CD and CE, the line BD will be greater than BE. Further, the triangles BAE and BAD have the sides AE and AD equal, and the side BA common, and the base BD greater than the base BE: therefore (*by the* 18. 1.) the angle BAD is greater than the angle BAE; adding therefore the equal angles CAD and CAE, the angles BAD and CAD will be greater than the angles CAE and BAE, *i. e.* the angle BAC.

PROPOSITION XXI.

A Theorem.

All the plain angles, that make one solid angle, are less than four right angles.



IF the plain angles BAC, BAD, and CAD, make the solid angle A, they will be less than four right angles. Draw the lines BC, BD, and CD, and you will have a pyramid, whose base is the triangle BCD.

Demonstration.

The solid angle at the point B, has the angles ABC and ABD greater than that of the base

base alone CBD. In like manner ACB and ACD are greater than BCD alone, and the angles ADC and ADB are greater than CDB alone. But all the angles of the base are equal to two right angles, therefore the angles ABC, ABD, ACB, ACD, ADC, and ADB, are greater than two right angles. And because all the angles of the three triangles BAC, BAD, and CAD, are equal to six right angles; taking away more than two right angles, there will remain less than four, for the angles made at the point A. But if the solid angle A consist of more than three plain angles, so that the base of the pyramid be a polygon, it may be divided into triangles; and the computation being made, you will find, that all the plain angles which make up the solid one, are always less than four right angles.

The USE.

'These two propositions shew when many 'plain angles may make up one solid one, 'which is often necessary in the treatises of 'cutting of Stones, and in the following pro'positions.

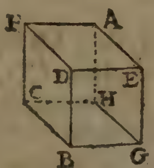
The 22. and 23. Propositions are of no use.

PRO-

PROPOSITION XXIV.

A THEOREM.

If a ſolid body be terminated by parallel planes, the oppoſite ſides will be ſimilar and equal parallelograms.



IF the ſolid AB be terminated by parallel planes, the oppoſite ſuperficies's will be ſimilar and equal parallelograms.

Demonſtration.

The parallel planes AC and *BE* are cut by the plane *FE*: therefore their common ſections are parallel, (*by the* 16.) and ſo likewiſe DF and AE; therefore AD will be a parallelogram. After the ſame manner I may demonſtrate, that AG, FB, CG, and the reſt, are parallelograms. I add, that the oppoſite parallelograms, *e. g.* AG and *FB*, are ſimilar and equal. The lines AE and EG are parallel to the lines FD and DB: therefore the angles AEG and *FDB* are equal, (*by the* 10.) Accordingly I may demonſtrate all the ſides and all the angles of the oppoſite parallelograms to be equal, therefore the parallelograms are ſimilar and equal.

PRO-

PROPOSITION XXV.

A Theorem.

If a Parallelepipedon be divided by a plane parallel to one of its planes, the two ſolid bodies which ariſe by that diviſion, will have the ſame proportion as their baſes.

IF the parallelepipedon AB be divided by the plane CD, which is parallel to the planes AF and BE, the ſolid AC will have the ſame proportion to BD, as the baſe AI to the baſe DG. Suppoſe the line AH which ſhews the hight of the figure to be divided into as many equal parts as you pleaſe; for example, ten thouſand; which we may take as indiviſibles, *i.e.* without reflecting upon the poſſibility of their being further ſubdivided. Suppoſe alſo ſo many ſuperficies's parallel to the baſe AI, as there are parts in the line AH; I have deſcribed only one OS: ſo that the ſolid AB be compounded of all thoſe ſuperficies's of the ſame thickneſs, as a Ream of paper is compounded of all its ſheets and quires laid one upon another. Tis evident that ſo the ſolid AC will be compounded of ten thouſand ſuper-

ſuperficies's equal to the baſe AI, (*by the preceding,*) and the ſolid DB will contain ten thouſand ſuperficies's equal to the baſe DG.

Demonſtration.

Every ſuperficies of the ſolid AC has the ſame proportion to any of the ſuperficies's of the ſolid DB, as the baſe AI to the baſe DG; becauſe they are every one of them equal to their baſes: therefore (*by the* 12. 5.) all the ſuperficies's of the ſolid AC, taken together, will have the ſame proportion to all thoſe of the ſolid DB, as the baſe AI to the baſe DG. But all the ſuperficies's of the ſolid AC make up the ſolid AC, which has no other parts but thoſe ſuperficies's; and all the ſuperficies's of the ſolid DB are nothing elſe but the ſolid DB; therefore the ſolid AC has the ſame proportion to the ſolid DB, as the baſe AI to the baſe DG.

The USE.

'This is *Cavalerius*'s demonſtration; which 'is very clear, provided it be uſed as it ought; 'and that the line, by which is meaſur'd the 'thickneſs of the ſuperficies's, be taken in the 'ſame reſpect in both the terms. I ſhall make 'uſe of it hereafter, to render ſome intricate 'and perplex'd demonſtrations more facil 'and clear.

PRO-

PROPOSITION XXVI.

A THEOREM.

A parallelepipedon is divided into two equal parts by the diagonal plane.

SUppose the parallelepipedon AB to be divided by the plane CD, drawn from one angle to another: I say it will be divided into two equal parts. Divide the line AE into as many parts as you pleaſe, and draw ſo many planes parallel to the baſe AF; each of thoſe places is a parallelogram equal to the baſe *AF*, (*by the* 24.)

Demonſtration.

All the parallelograms, that can be drawn parallel to the baſe *AF*, are divided into two equal parts by the plane CD; for the triangles which are form'd on both ſides the plane CD, have their baſe common, in each equal to CD, and their ſides equal, being thoſe of a parallelogram. But 'tis evident, that the parallelepipedon is nothing elſe but thoſe parallelograms, which are each divided into two equal triangles: therefore the parallelepipedon is divided into two equal parts by the plane CD.

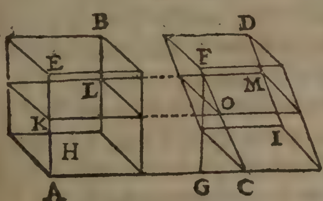
'The

The 27. and 28. Propositions are of no use according to this way of demonstrating.

PROPOS. XXIX, XXX, XXXI.

A THEOREM.

Parallelepipedons of the same hight, having the same or equal bases, are equal.



If the Parallelepipedons AB and CD be of the same hight according to the perpendiculars AE and FG, and have the same or equal bases AH and CI, they will be equal. Suppose the two bases to be set upon the same plane; since their perpendiculars are equal, the bases EB and FD will be in the same plane, which will be parallel to the plane of the bases AH and CI. Suppose then the line AE or FG to be divided into as many equal parts as you please, *e. g.* ten thousand, and according to them so many superficies's or planes drawn of the same thickness: I have describ'd only one for all, as K or M. Each superficies will form in these solids a parallel plane, similar and equal to the base, (*by the 24.*) as KL, OM; and

and there will be as many in one ſolid as in the other; becauſe their thickneſs, which I take perpendicularly according to their *reſpective* hights, is equal.

Demonſtration.

The baſe AH has the ſame proportion to the baſe CI, as each plane KL to OM. But they being equal in number in both, all the antecedents (*by the* 15.5.) will have the ſame proportion to all the conſequents, *i. e.* the whole ſolid AB to the whole ſolid CD, as the baſe AH to the baſe CI. But 'tis ſuppos'd that the baſes are equal, therefore the ſolids are equal.

Coroll. To find the ſolidity of a parallelepipedon, 'tis uſual to multiply the baſe by the hight taken perpendicularly, becauſe that perpendicular ſhows how many ſuperficies's equal to the baſe are contain'd in it. As for example, if I take a foot for my indiviſible meaſure, *i. e.* which I will not afterwards ſubdivide; if the baſe contain twelve feet ſquare, and the perpendicular hight ten, I ſhall have an hundred and twenty cubick feet for the ſolidity of the body AB. For the hight containing ten feet, I may make ten parallelograms equal to the baſe, having each a foot in thickneſs; but the baſe with one foot in thickneſs makes twelve cubick feet: the whole therefore will make an hundred and twenty, if the hight contain ten feet.

PROP.

PROPOSITION XXXII.

A Theorem.

Parallelepipedons of the same hight are in the same proportion as their bases.

I Have prov'd this proposition in the preceding, demonstrating, that the parallelepipedon *A B* has the same proportion to the parallelepipedon *CD*, as the base *AH* to the base *CI*. *(See fig. preced.)*

Coroll. Parallelepipedons that have equal bases, are in the same proportion as their hights. As the parallelepipedons *AB* and *AL*, whose perpendicular hights are *AK* and *AE*. For if you divide the hight *AK* into as many aliquot parts as you please, and *AE* into as many as it contains equal to the former, and draw, according to each part, planes parallel to the base; as many as *AE* contains of the aliquot parts of *AK*, so many will the solid *AB* contain of the superficies's equal to the base, which are the aliquot parts of the solid *AL*; therefore (*by defin.* 5. 5.) the solid *AB* will have the same proportion to the solid *AL*, as the hight *AE* to the hight *AK*.

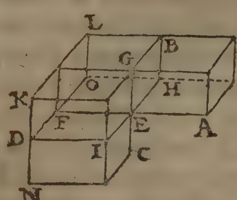
The U S E.

'The three preceding propoſitions contain
'almoſt all the ways of meaſuring parallelepi-
'pedons, and may be eſteem'd as firſt prin-
'ciples for that purpoſe. 'Tis after the ſame
'manner alſo that we take the dimenſions of
'the ſolidity of Walls, by multiplying their
'baſes by their hights.

PROPOSITION XXXIII.

A THEOREM.

Similar parallelepipedons are in the triplicate proportion of their homologous ſides.



IF the parallelepipedons AB and CD be ſimiliar, *i. e.* if all the planes of one be like thoſe of the other; and all their angles equal, ſo that they may be plac'd in a right line, *i. e.* that AE and EF, HE and EI, GE and EC, may make right lines; and AE has the ſame proportion to EF as HE to EI, and as GE to EC: I ſay, that here are four ſolids in continual proportion according to the proportion of the ſide EA to that, which is homologous to it, EF or DI.

De-

Demonſtration.

The parallelepipedon AB has the ſame proportion to EL of the ſame hight, as the baſe AH to the baſe EO, (*by the* 32.) But the baſe AH has the ſame proportion to the baſe EO, as AE to EF, (*by the* 1. 6.) In like manner, the proportion of the ſolid EL to the ſolid EK, is the ſame with that of the baſe EO to the baſe ED, *i. e.* that of HE to EI. And laſtly, the ſolid EK has the ſame proportion to the ſolid EN, as the hight GE to the hight EC, (*by the coroll. of the* 32.) or (taking the line EF for their common hight) as the baſe GI to the baſe CI, *i. e.* as GE to EC. But the proportion of AE to EF, of HE to EI, and of GE to EC, was ſuppos'd to be the ſame; and conſequently, the ſolid AB has the ſame proportion to EL as EL to EK, and as EK to CD. Therefore (*by defin.* 11. 5.) the proportion to AB to CD will be the triplicate proportion of that of AB to EL, or of AE to its homologous ſide EF.

Coroll. If four lines be in continual proportion, the parallelepipedon deſcrib'd upon the firſt, has the ſame proportion to another ſimilar parallelepipedon deſcrib'd upon the ſecond, as the firſt to the fourth; for the proportion of the firſt to the fourth, is the triplicate proportion of that of the firſt to the ſecond.

The USE.

‘ You may perceive by this propoſition that

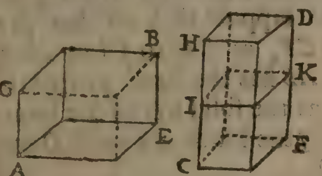
'that famous problem of the duplication of the 'Cube, propos'd by the Oracle, consists in finding two middle terms in continual proportion. 'For if you make the side of the first cube the 'first term, and the double of that the fourth; 'having found twe middle proportionals, the 'cube describ'd upon the first line will have the 'same proportion to that describ'd upon the second, as the first line to the fourth, *i.e.* as one 'to two. By this proposition also may be corrected their error, who fancy similar solids to 'have the same proportion as their sides; as if a 'cube of one foot in length was the half of a 'cube two foot long; when indeed it is but the 'eighth part thereof. This is likewise the foundation of the Rule concerning the size of the 'bores of Canons; and is applicable not only to 'bullets, but to all sorts of similar bodies. For 'example; should a man, about to build a Navy, and resolving to retain the same proportion in all his Vessels, reason thus with him 'self; If a ship of an hundred tun require fifty 'foot in Keel, another of two hundred tuns 'ought to have an hundred foot in Keel; he 'would be guilty of a great mistake: for instead of making a Vessel twice as large as the 'former, he would make one eight times so 'much. He ought to assign to the second Vessel somewhat less than sixty three feet.

PROP.

PROPOSITION XXXIV.

A Theorem.

Equal parallelepipedons have their bases's and hights reciprocal, and those that have their bases and hights reciprocal are equal.



If the parallelepipedons *AB* and *CD* be equal, their bases and hights will be reciprocal, *i. e.* the base *AE* will have the same proportion to the base *CF*, as the hight *CH* to the hight *AG*. Having made *CI* equal to *AG*, draw the plane *IK* parallel to the base *CF*.

Demonstration.

The parallelepipedon *AB* has the same proportion to *CK*, being of the same hight, as the base *AE* to the base *CF*, (*by the* 32.) But as *AB* to *CK*, so is *CD* to the same *CK*, because *AB* and *CD* are equal; and as *CD* to *CK*, which have both the same base, so is the hight *CH* to the hight *CI*, (*by the Coroll. of the* 32.) therefore as the base *AE* to the base *CF*, so is the hight *CH* to the hight *CI* or *AG*.

I add, that if the baſe *AE* has the ſame proportion to the baſe *CF* as the hight *CH* to the hight *AG*, the ſolids *AB* and *CD* will be equal.

Demonſtration.

AB has the ſame proportion to *CK*, being of the ſame hight, as the baſe *AE* to the baſe *CF*, (*by the* 32.) Alſo the hight *CH* has the ſame proportion to the hight *CI* or *AG*, as *CD* to *CK*: But we ſuppoſe that *AE* has the ſame proportion to *CF*, as *CH* to *CI* or *AG*; therefore the ſolid *AB* has the ſame proportion to the ſolid *CK* as the ſolid *CD* to the ſame *CK*, and conſequently the ſolids *AB* and *CD* are equal, (*by the* 9. 5.)

The USE.

'This Reciprocation of the baſes and hights 'makes the ſolid very eaſie to be meaſur'd. And 'the propoſition ſeems to bear ſome analogy to 'the 14. *Prop. of the* 6. which aſſerts, That e'quiangular and equal parallelograms have 'their ſides reciprocal; and the practice of 'the Rule of three may be demonſtrated from 'both.

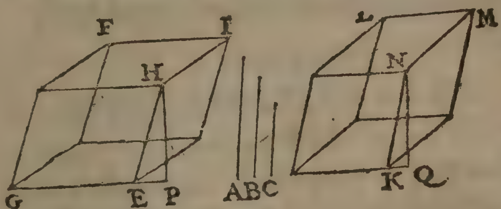
The 35. *Prop. may be omitted.*

PRO-

PROPOSITION XXXVI.

A THEOREM.

If three lines be in continual proportion, a parallelepipedon made of those three lines is equal to an equiangular parallelepipedon, which has all its sides equal to the middle line.



IF the lines A, B, C, be in continual proportion, the parallelepipedon EF made of those three lines, the side FI being equal to the line A, HE equal to B, and HI equel to C, is equal to the equiangular parallelepipedon KL, whose sides LM, MN, and KN, are each of them equal to the line B. From the points H and N draw the lines HP and NQ perpendicular to the planes of the bases; which lines will be equal, because the solid angles E and K are suppos'd equal, (so that if they could penetrate, neither would exceed the other,) and the lines EH and

and KN are alſo ſuppos'd equal. Therefore the hights HP and NQ are equal.

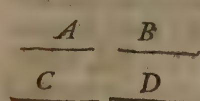
Demonſtration.

There is the ſame proportion of A to B, or of FI to LM, as of B to C, or LM to HI: Therefore the parallelogram FH contain'd under FI and IH is equal to the parallelogram LN contain'd under LM and MN both equal to B, (*by the* 16. 6.) therefore the baſes are equal. But the hights HP and NQ are alſo equal; therefore (*by the* 31.) the parallelepipedons are equal.

PROPOSITION XXXVII.

A Theorem.

If four lines be proportional, the parallelepipedons deſcrib'd upon thoſe lines are proportional: and if the ſimilar parallelepipedons be proportional, their homologous ſides will be alſo proportional.



If the line A has the ſame proportion to B as C to D, the ſimilar parallelepipedons, whoſe homologous ſides are the lines A,B,C,D, will be in the ſame proportion.

Demonſtration.

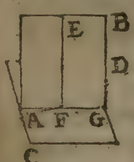
The parallelepipedon A is in the triplicate proportion to the parallelepipedon B, of that of

of the line *A* to the line *B*, or that of the line *C* to the line *D*. But the parallelepipedon *C* to the parallelepipedon *D* is alſo in the triplicate proportion of that of the line *C* to the line *D*, (*by the* 33.) Therefore the parallelepipedon *A* has the ſame proportion to the parallelepipedon *B*, as the parallelepipedon *C* to the parallelepipedon *D*.

PROPOSITION XXXVIII.

A THEOREM.

If two planes be perpendicular to each other, a perpendicular drawn from a point in one of the planes to the other will fall upon the common ſection.



IF, the planes AB and CD being perpendicular to each other, you draw from the point E in the plane AB a line perpendicular to the plane CD, it will fall upon the common ſection of the planes. Draw EF perpendicular to the common ſection AG.

Demonſtration.

The line EF, perpendicular to AG, the common ſection of the planes, which are ſuppos'd to be perpendicular, will be perpendicular to the plane CD, (*by defin.* 3.) and becauſe two lines

lines cannot be drawn from the point E perpendicular to the plane CD, (*by the* 13.) every perpendicular will fall upon the common ſection AG.

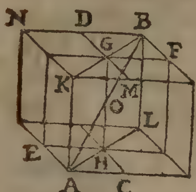
The U S E.

'This propoſition ought to have follow'd next 'after the 17*th*. becauſe it reſpects ſolids in 'general. 'Tis of uſe to us in the Treatiſe of '*Aſtrolabes*, to prove that in the *Analemma* all 'the circles, perpendicular to the *Meridian*, 'ought to be markt by right lines.

PROPOSITION XXXIX.

A Theorem.

If in a parallelepipedon be drawn two planes, which divide the oppoſite ſides into two equal parts, their common ſection and the diameter will alſo divide each other into two equal parts.



SUppoſe the oppoſite ſides of the parallelepipedon AB to be divided into two equal parts by the planes CD and EF, their common ſection GH and the diameter BA will equally divide each other at the point O. Draw the lines BG, GK, AH and HL. I ſhall prove firſt, that the two

two firſt of theſe, BG and GK, (and ſo like-wiſe AH and HL,) make but one right line. For the triangles DGB and KMG have their ſides DB and KM equal, becauſe they are the halves of equal ſides; as alſo GD and GM. Further, DB and KM being parallel, the alternate angles BDG and GMK will be equal, (*by the* 29. 1.) and therefore (*by the* 4. 1.) the triangles DBG and KGM will be equal in all reſpects, and conſequently the angles BGD and KGM: and [*by the coroll. of the* 15. 1.] BG and GK make but one right line, as alſo LH and HA: therefore ALBK is one plane, in which are found both the diamater AB, and the common ſection of the planes GH. The plane ALBK cutting the parallel planes AN and CD, their common ſections GH and AK will be parallel: And [*by the* 2. 6.] BG will have the ſame proportion to GK, as BO to OA; and therefore [*by the* 18. 5.] as BK to GK, ſo BA to BO; and [*by the* 4. 6.] ſo GH or AK to OG. But BK is double to BG, therefore BA is double to BO and AK, equal to GH, double to GO. Therefore the lines GH and AB divide each other equally at the point O.

Coroll. 1. All the diameters are divided at the point O.

Coroll. 2. Here we may add ſome Corollaries, which depend upon divers propoſitions. As for example, that triangular priſms of the ſame

ſame hight are in the ſame proportion as their baſes: For the parallelepipedons, of which they are the halves, are [*by the* 32.] in the ſame proportion as their baſes: Therefore the halves of their baſes, and the halves of the parallelepipedons, *i. e.* the priſms, will be in the ſame proportion.

Coroll. 3. Polygon priſms of the ſame hight are alſo in the ſame proportion as their baſes, becauſe they may be reſolv'd into triangular ones, each of which will have the ſame proportion as their baſes.

Coroll. 4. The reſt of the propoſitions concerning parallelepipedons are alſo applicable to priſms: as for example, that equal priſms have their hights and baſes reciprocal; and that ſimilar priſms are in the triplicate proportion of that of their homologous ſides.

The USE.

'This propoſition may help us to find out the 'center of Gravity in parallelepipedons; and 'to demonſtrate ſome other propoſitions in the 'thirteenth and fourteenth books of *Euclid*.

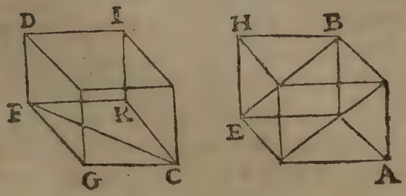
PROP.

PROPOSITION XL.

A Theorem.

A Priſm, that has a parallelegram for its baſe double to the triangular baſe of another priſm, and of the ſame hight, is equal to it.

LET ABE and CDG be two triangular priſms, of the ſame hight; and the baſe of one the parallelogram AE, double to the triangle FGC, the baſe of the other priſm: I ſay theſe priſms are equal. Suppoſe the parallelepipedons AH and GI were compleated.



Demonſtration.

'Tis ſuppos'd, that the baſe AE is double to the triangle FGC, but the parallelogram GK is double to the ſame triangle, [*by the* 34. 1] therefore the parallelograms AE and GK are equal;

equal; and consequently the parallelepipedons AH and GI, having the same bases and the same hights, are equal; and therefore the prisms that are the halves, [*by the* 26.] will be likewise equal.

THE

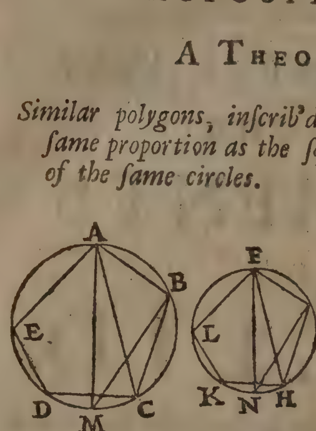
THE TWELFTH BOOK OF THE ELEMENTS OF EUCLID.

'*EUclid*, after having in the preceding Books 'deliver'd the general principles of ſolid 'bodies, and explain'd the manner of meaſuring 'the moſt regular of them, that is, ſuch as are 'terminated by plain ſuperficies's; treats in this 'of ſuch bodies as are contain'd in ſuperfices's 'that are crooked, as the Cylinder, Cone, and 'Sphere: comparing one with the other, and 'giving rules, relating both to their ſolidity, 'and the manner of taking their dimenſions. 'The Book is of exceeding great uſe, becauſe 'in it we find the principles upon which the 'moſt learned *Mathematicians* have built ſo 'many famous demonſtrations concerning the 'Cylinder, the Cone, and the Sphere.

PROPOSITION I.

A THEOREM.

Similar polygons, inscrib'd in circles, are in the same proportion as the squares of the diameters of the same circles.



IF the polygons ABCDE, and FGHKL, inscrib'd in circles, be similar, they will be in the same proportion as the squares of the diameters AM, FN. Draw the lines BM, GN, AC, and FH.

Demonstration.

'Tis suppos'd that the polygons are similar, that is to say, that the angles B and G are equal, and that AB has the same proportion to BC as FG to GH: from whence I infer, [*by the* 6.6.] that the triangles ABC and FGH are equiangular, and that the angles ACB and FHG are equal: so that likewise [*by the* 21. 3] the angles AMB and FNG are equal. But the angles ABM and FGN, being in a semicircle, are right angles, [*by the* 31. 3.] and consequently, the triangles ABM and FGN are equiangular.

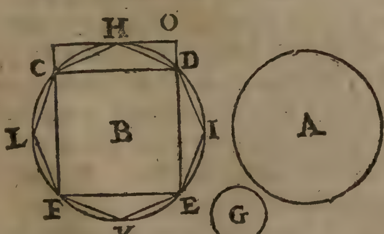
There-

Therefore [*by the* 4. 6.] AB has the ſame proportion to FG, as AM to FN: and [*by the* 22. 6.] if two ſimilar polygons be diſcrib'd upon AB and FG, as thoſe that are propos'd; and two other ſimilar polygons upon AM and FN, which ſhall be two ſquares; the polygon ABCDE will have the ſame proportion to the polygon FGHKL, as the ſquare of AM to the ſquare of FN.

" This propoſition is neceſſary to demon-
" ſtrate that which follows.

LEMMA.

If a certain quantity be leſs than a circle, a regular polygon may be inſcrib'd in the ſame circle greater than that quantity.



Suppoſe the figure A ' to be leſs ' than the circle B; a regular polygon may be ' inſcrib'd in 'the ſame circle, which ſhall be greater than the 'figure A. Let the figure G be the difference 'between the figure A and the circle, ſo that the figures

'figures A and G taken togother, may be equal 'to the circle B. Inſcribe in the circle B the 'ſquare CDEF, [*by the* 6. 4.] and if the ſquare 'be greater than the figure A, we ſhall have 'what we wanted. If it be leſs, divide the four 'quarters of the circle CD, DE, EF, and FC, 'each into two equal parts at the points H,I,K, 'L, that ſo you may have an octogon. But if 'the octogon be ſtill leſs than the figure A,ſub-'divide its archs, and you will have a polygon 'of ſixteen ſides, afterwards of thirty two, and 'then of ſixty four. I ſay, at length you will have 'a polygon greater than the figure A, *i.e.* a po-'lygon whoſe difference from the circle is leſs 'than that of the figure A, that is leſs than the 'figure G.

Demonſtration.

'The inſcrib'd ſquare is more than half of the 'circle, being half of the ſquare deſcrib'd about 'the circle; and in deſcribing the octogon you 'take more than half of the Remainder, *i. e.* of 'the four ſegments CHD, DIE, EKF, and CLF, 'For the triangle CHD is the half of the rect-'angle CO, [*by the* 34. 1.] therefore it is more 'than half of the ſegment CHD; and the ſame 'may be ſaid of all the other arches. In like 'manner, in deſcribing the polygon of ſixteen 'ſides, you take more than half of what was left 'of the circle; and ſo in all the others. There-'fore you will leave at laſt a leſs quantity than 'G. For 'tis evident, that two unepual quanti-

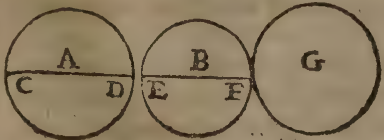
ties

'ties being propos'd, if you take away more 'than half of the greater, and afterwards more 'than half of what remains, and again more than 'half of what is ſtill left behind ; at length that 'which remains will be leſs than the ſecond 'quantity. Suppoſe the ſecond quantity to be 'contain'd in the firſt an hundred times : 'tis 'evident, that dividing the firſt into an hundred 'parts,in ſuch ſort, that the firſt part may have 'a greater proportion to the ſecond than two to 'one ; the laſt will be leſs than the hundredth 'part : ſo that at laſt you will obtain a polygon, 'which will be leſs exceeded by the circle, than 'the circle exceeds the figure A ; that is to ſay, 'that what will remain of the circle, when the 'polygon is taken away, will be leſs than G. 'Therefore the polygon will be greater than the 'figure A.

PROPOSITION II.

A THEOREM.

Circles are in the ſame proportion as the ſquares of their diameters.



I Prove, that the circles A and B are in the ſame proportion, as the

the ſquares of CD and EF. Suppoſe the figure G to have the ſame proportion to the circle B, as the ſquare of CD to the ſquare of EF: if the figure G be leſs than the circle A, [*by the preceding Lemma,*] a regular polygon may be inſcrib'd in the circle A greater than G. Let a ſimular regular polygon be alſo inſcrib'd in the circle B.

Demonſtration.

The polygon of the circle A will have the ſame proportion to the polygon of B, as the ſquare of CD to the ſquare of EF, *i. e.* the ſame as G to the circle B; but the quantity G is leſs than the polygon inſcrib'd in A: accordingly therefore [*by the* 14.5.] the circle B muſt be leſs than the polygon inſcrib'd in it, which is manifeſtly falſe. It muſt therefore be granted that the figure G, being leſs than the circle A, cannot have the ſame proportion to the circle B, as the ſquare of CD to the ſquare of EF; and conſequently, that the circle A cannot have a greater proportion to the circle B, than the ſquare of CD to the ſquare of EF: nor can it be ſaid to have a leſs; for then the circle B would have a greater proportion to the circle A, and the ſame demonſtration would be applicable to it.

Coroll. 1. Circles are in the duplicate proportion of that of their diameters; becauſe the ſquares being ſimilar figures, are in the duplicate

cate proportion of that of their ſides, [*by the* 20. 6.]

Coroll. 2. Circles are in the ſame proportion as the ſimilar polygons, that are inſcrib'd in them.

Coroll. 3. This ought to be well obſerv'd as a general rule: When ſimilar figures, being inſcrib'd in others, ſo that they may approach ſtill nearer and nearer to them, and at laſt degenerate into the figures themſelves, are in the ſame proportion; the figures that contain them are alſo in the ſame proportion. What I would ſay is this; That ſimular regular polygons, inſcrib'd in divers circles, are always in the ſame proportion as the ſquares of the diameters; and being made of more ſides, ſo as to approach ſtill nearer and nearer to the circles, they ſtill retain the ſame proportion; and the circles themſelves are in the ſame proportion as the ſquares of their diameters. This manner of meaſuring round bodies, by inſcribing in them others, is of great uſe.

The USE.

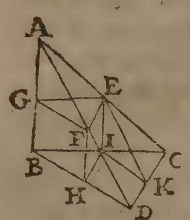
'This being a very general Propoſition, en-'ables us to argue about circles in the ſame man-'ner as we do of ſquares. For example, we ſay '[*in the* 47. 1.] that in a rectangle triangle the 'ſquare of the baſe alone is equal to the ſquares 'of both the ſides taken together. We may ſay 'the ſame of circles, *i. e.* That the circle, de-

'ſcrib'd upon the baſe of a rectangle triangle, is 'equal to the circles, whoſe diameters are the 'ſides. And in the ſame manner may we aug-'ment or diminiſh circles, according to what 'proportion we pleaſe. We prove alſo by it in '*Opticks*, that the light decreaſes in the dupli-'cate proportion of that of the diſtances of the 'lucid bodies.

PROPOSITION III.

A Theorem.

Every Pyramid, whoſe baſe is triangular, may be divided into two equal priſms, which make up more than half of the pyramid; and into two equal pyramids.



In the pyramid ABCD may be found two equal priſms, EBFI, and EHKC, which will be greater than half the pyramid. Divide the ſix ſides of the pyramid equally at the points G,F,E,I, H,K, and draw the lines EG, GF, FE, EI, HI, FH, IK, and EK.

Demonſtration.

In the triangle ABD, AG has the ſame proportion to GB as AF to FD, becauſe AB and

AD

AD are equally divided in G, and F; therefore [*by the* 2.6.] GF and BD are parallels; and GF will be the half of BD, *i. e.* equal to BH. In like manner, GE and BI, FE and HI, will be paarllels, and equal: and [*by the* 15.11.]the planes GFE and BHI will be parallel, and consequently EBFI will be a prism. The same may be said of the figure HEKF,which will be also a prism equal to the other, [*by the* 40.11.] the parallelogram base HIKD being double the triangular BHI, [*by the* 41. 1.]

Secondly, I say, the pyramids AEFG, and ECKI, are similar and equal.

Demonstration.

The triangles AFG and FDH are equal, (*by the* 8. 1.) as also FDH and EIK; and likewise AGE, and EIC, and so of all the other triangles of the pyramids: therefore the pyramids are equal, (*by defin.* 10.11.] They are also similar to the great pyramid ABDC: for the triangles AGE, and ECI are similar, (*by the* 2.6.) the lines GE and BC being parallels; and the like may be demonstrated of all the other triangles of the lesser pyramids.

Lastly, I say the prisms are more than half of the first pyramid. For if each was equal to one of the lesser pyramids, both would be equal to the half of the greater pyramid. But they are each of them greater than one of those pyramids; as the prism GHE contains the pyramid

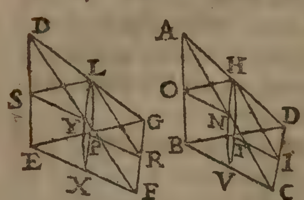
GBHI

GBHI,and ſomewhat more; and that pyramid is equal and ſimilar to the others, having all their triangles equal and ſimilar to thoſe of the pyramid AGFE,as may be eaſily prov'd by the paralleliſm of their ſides: from whence I infer, that the two priſms taken together are greater than the two pyramids, and conſequently greater than half of the great pyramid.

PROPOSITION IV.

A THEOREM.

If two triangular pyramids of the ſame hight be divided into two priſms and two pyramids, and the latter pyramids ſubdivided after the ſame manner; all the priſms of one pyramid will have the ſame proportion to all thoſe of the other, as the baſe of one pyramid to the baſe of the other.



IF the two pyramids ABCD, DEFG, of the ſame hight, and having triangular baſes, be divided into two priſms and two pyramids, according to the method laid down in the third propoſition; and the two leſſer pyramids be ſubdivided after the ſame manner, and ſo

ſo in order, that, having made as many diviſions of one as of the other, you have the ſame number of priſms in both; I ſay, that all the priſms of one will have the ſame proportion to all the priſms of the other, as their baſes.

Demonſtration.

The pyramids being of the ſame hight, the priſms, produc'd by the firſt diviſion, will have alſo the ſame hight, becauſe they have each the half of that of their pyramids. But priſms of the ſame hight are in the ſame proportion as their baſes, (*by the coroll. of the* 39. 11.) The baſes BTV and EPX are ſimilar to the baſes BDC and EGF; and having for their ſides the half of thoſe great baſes, they can make but the fourth part of them, but they are in the ſame proportion as the great baſes are; therefore the firſt priſms will have the ſame proportion as the great baſes. After the ſame manner I may prove that the priſms produc'd by the ſecond diviſion, *i.e.* of the leſſer pyramids, will be in the ſame proportion as the baſes of thoſe leſſer pyramids, which are in the ſame proportion as the great baſes. Therefore all the priſms of one have the ſame proportion to all the priſms of the other, as the baſe to the baſe.

The USE.

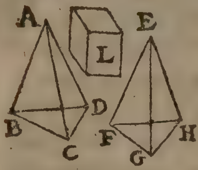
"These two propoſitions are neceſſary to "compare pyramids together, and to take their "dimenſions.

PROP.

PROPOSITION V.

A Theorem.

Triangular pyramids of the ſame hight are in the ſame proportion as their baſes.



THE pyramids ABCD and EFGH are in the ſame proportion as their baſes. For if they were not, one of them, *e. g.* ABCD, would have a greater proportion to the pyramid EFGH, than the baſe BCD to the baſe FGH; ſo that a quantity leſs than ABCD would have the ſame proportion to the pyramid EFGH, as the baſe BCD to the baſe FGH. Divide the pyramid ABCD after the manner of the third propoſition; divide alſo the pyramids, that reſult from that firſt diviſion, into two priſms and two pyramids, and thoſe again into two other priſms, continuing the diviſion as long as there ſhall be occaſion. Since the priſms of the firſt diviſion are more than the half of the pyramid ABCD, (*by the* 3.) and the priſms of the ſecond diviſion more than half the remainder, *i.e.* of the two leſſer pyramids, and thoſe of the third diviſion ſtill more than the half of what is left; it is evident, that ſo

ſo many diviſions may be made,that that which remains ſhall be leſs than the exceſs of the pyramid ABCD above the quantity L, that is, that all the priſms taken together ſhall be greater than the quantity L. Make as many diviſions of the pyramid EFGH, ſo that you may have as many priſms as there are in ABCD.

Demonſtration.

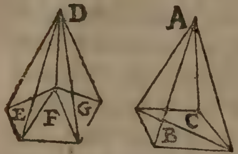
The priſms of ABCD have the ſame proportion to the priſms of EFGH, as the baſe BCD to the baſe FGH : but the proportion of the baſe BCD to the baſe FGH is the ſame with that of the quantity L to the pyramid EF GH : therefore the priſms of ABCD have the ſame proportion to the priſms of EFCH, as the quantity L to the pyramid EFGH. But alſo the priſms of ABCD are greater than the quantity L : therefore (*by the* 14. 5.) the priſms contain'd in the pyramid EFGH would be greater than the ſame pyramid EFGH, which is evidently falſe, becauſe the part cannot be greater than the whole. Therefore it muſt be granted, that no quantity leſs than one of the pyramids can have the ſame proportion to the other as the baſe to the baſe, and conſequently neither of the pyramids can have a greater proportion to the other than the baſe to the baſe.

PROP.

PROPOSITION VI.

A THEOREM.

All sorts of pyramids, of the same hight, have the same proportion as their bases.



THE pyramids ABC and DEFG, of the same hight, are in the same proportion as the bases BC and EFG. Divide the bases into triangles.

Demonstration.

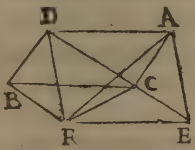
The triangular pyramids AB and DE, being of the same hight, are in the same proportion as their bases, (*by the* 5.) So also the triangular pyramids AC and DF are in the same proportion as their bases. Therefore the pyramid ABC has the same proportion to the pyramid DEF as the base BC to the base EF, (*by the* 12. 5.) Further, since the pyramid DEF has the same proportion to the pyramid ABC, as the base EF to the base BC; and again, the pyramid DG has the same proportion to the pyramid ABC, as the base G to the base BC; the pyramid DEFG will also have the same proportion to the pyramid ABC, as the base EFG to the base BC.

PROP.

PROPOSITION VII.

A THEOREM.

Every pyramid is the third part of a priſm, being upon the ſame baſe, and of the ſame hight.



SUppoſe firſt the triangular priſm AB be propos'd: I ſay, a pyramid, having one of the triangles ACE or BDF for its baſe, and being of the ſame hight, as the pyramid ACEF, will be the third part of the priſm. Draw the three diagonals AF, DC, FC, of the three parallelograms.

Demonſtration.

The priſm is divided into three equal pyramids, ACFE, ACFD, and CFBD; therefore each will be the third part of the priſm. The two firſt, having for their baſes the triangles AEF and AFD, which (*by the* 34.1.) are equal, and for their hight, the prependicular drawn from the top C to the plane of their baſes AF, will be equal, (*by the preceding*,) The pyramids ACFD, and CFBD, which for their baſes have the equal triangles ADC and DCB, and the ſame top F, will be alſo equal, (*by the preceding*.) Therefore one of thoſe pyramids, *e. g.* AFCE,

AFCE, having the ſame baſe BDF with the priſm, and the ſame hight, which is the perpendicular drawn from the point F to the plane of the baſe ACE, is the third part of the ſame priſm. If the priſm be a polygon, it muſt be divided into divers triangular priſms; and the pyramid, which has the ſame baſe, and the ſame hight, will be alſo divided into as many triangular pyramids; each of which will be the third part of its priſm. Therefore (*by the* 12. 5.) the polygon pyramid will be the third part of the polygon priſm.

PROPOSITION VIII.

A Theorem.

Similar pyramids are in the triplicate proportion of that of their homologous ſides.

IF the pyramids be triangular, compleat the priſms, which will be alſo ſimilar, becauſe they will have certain planes the ſame with thoſe of the pyramids. But the ſimilar priſms are in the triplicate proportion of their homologous ſides, [*by Coroll.* 4. *of the* 39. 11.] therefore the pyramids, which (*by the preceding*] are the third parts of the priſms, will be in the triplicate proportion of that their homologous ſides.

If the pyramids be polygons, they muſt be reduc'd to triangular pyramids.

PROP.

PROPOSITION IX.

A THEOREM.

Equal pyramids have their hights and bases reciprocal, and those that have their heights and bases reciprocal are equal.

IF two equal triangular pyramids be propos'd, make prisms upon the same base, and of the same hight. Since every prism is triple his pyramid, (*by the* 7.) they will also be equal. But equal prisms have their bases and hights reciprocal, (*by Coroll.* 4. *of the* 39. 11.) therefore the bases and hights of the pyramids, which are the same with those of the prisms, will be also reciprocal.

Secondly, if the bases and hights of the pyramids be reciprocal, the prisms will be equal, as also the pyramids, which are the third parts of the prisms.

If the pyramids propos'd be polygons, they must be reduc'd to triangular pyramids.

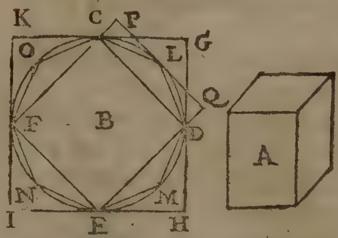
Coroll. Other propositions may be made concerning pyramids: as for example; That pyramids of the same hight, are in the same proportion as their bases; and those that have the same bases, are in the same proportion as their hights.

The U S E.

‘From theſe propoſitions is drawn the manner of meaſuring pyramids, which is, by multiplying their baſes by the third part of their hights. Other propoſitions may alſo be made, as, That if a priſm be equal to a pyramid, the baſes and the hight of the priſm, with the third part of the hight of the pyramid, will be reciprocal; which is as much as to ſay, that if the baſe of the pyramid has the ſame proportion to the baſe of the priſm, as the hight of the priſm to the third part of the hight of the pyramid, the priſm and the pyramid will be equal.

A LEMMA.

If a quantity leſs than a Cylinder be propos'd, a polygon priſm may be inſcrib'd in the Cylinder grater than that quantity.



IF the quantity A be leſs than ‘the cylinder, whoſe baſe is the circle B, a polygon priſm may be inſcrib'd in the cylinder greater than the quantity A. The

ſquare

'ſquare CDEF, inſcrib'd in, and GHIK circum-'ſcrib'd about the circle. CLDMENFO is an 'octogon inſcrib'd. Draw the tangent PLQ: and 'ſuppoſe you had ſo many priſms as there are 'polygon baſes, and all of the ſame hight with 'the cylinder. That which has the circumſcrib'd 'ſquare for its baſe, will encompaſs the cylin-'der; and that whoſe baſe is the inſcrib'd ſquare, 'will be alſo inſcrib'd in the cylinder.

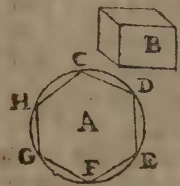
Demonſtr. Priſms of the ſame hight are in 'the ſame proportion as their baſes, (*by coroll.* 3. '*of the* 39. 11.) and the inſcrib'd ſquare being 'the half of that which is circumſcrib'd, its 'priſm will be the half of the other, and there-'fore more than the half of the cylinder. And 'making the priſm with the octogon baſe, you 'take away more than half of what remain'd of 'the cylinder, after the priſm of the inſcrib'd 'ſquare was taken from it, becauſe the triangle 'CLD is the half of the rectangle CQ. And be-'cauſe priſms of the ſame hight are in the ſame 'propotion as their baſes, the priſm, whoſe 'baſe is the triangle CLD, will be the half of 'the priſm, which for its baſe has the rectangle 'DCPQ: it will therefore be more than the 'half of that part of the cylinder, whoſe baſe is 'the ſegment DLC. The ſame may be ſaid of 'all the other ſegments. After the ſame manner 'I may demonſtrate, that making a polygon 'priſm of ſixteen ſides, I take away more than

'half of what remains of the cylinder, after the 'octogon prifm is taken from it: fo that there 'will remain at laft a part of the cylinder, lefs 'than the excefs of the cylinder above the quan- 'tity A. We fhall have therefore a prifm in- 'fcrib'd in the cylinder, which fhall be lefs ex- 'ceeded by the cylinder than the quantity A. '*i.e.* which fhall be greater than the quantity A. 'The fame way of arguing will hold of the py- 'ramids infcrib'd in a cone.

PROPOSITION X.

A Theorem.

A Cone is the third part of a cylinder, having the fame bafe, and being of the fame hight.



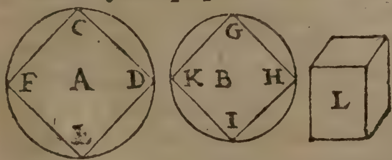
If a cone and a cylinder have the circle A for their bafe, and be of the fame hight, the cylinder will be triple the cone. For if the proportion of the cylinder to the cone was greater than the triple proportion, the quantity B lefs than the cylinder would have the fame proportion to the cone as three to one: and (*by the preceding Lemma*) a polygon prifm may be infcrib'd in the cylinder greater than the quantity B. Suppofe that which has for its bafe the polygon CDEFGH to be fuch an one. Make alfo upon the fame bafe a pyramid infcrib'd in the cone. *De-*

Demonſtr. The cylinder, the cone, the priſm, and the pyramid, are of the ſame hight; therefore the priſm is the triple of the pyramid, (*by the* 7.) But the quantity B is alſo the triple of the cone; therefore the priſm has the ſame proportion to the pyramid, as the quantity B to the cone: and (*by the* 14. 5.) the priſm being greater than the quantity B, the pyramid would be greater than the cone, in which it is inſcrib'd, which is impoſſible.

But if it be ſaid, that the cone has a greater proportion to the cylinder than one to three, the ſame method may be made uſe of to demonſtrate the contrary.

PROPOSITION XI.
A THEOREM.

Cylinders and Cones of the ſame hight are in the ſame proportion as their baſes.



LET two cones, or two cylinders, of the ſame hight, be propos'd, having for their baſes the circles A and B; I ſay, they are in the ſame proportion as their baſes. For if not, one of them, *e. g.* the cylinder A would have a greater proportion to the

the cylinder B, than the baſe A has to the baſe B; ſuppoſe then that the quantity L, leſs than the cylinder A, has the ſame proportion to the cylinder B, as the baſe A to the baſe B. Therefore a polygon priſm may be inſcrib'd in the cylinder A, which ſhall be greater than the quantity L. Suppoſe it that therefore, whoſe baſe is the polygon CDEF; and inſcribe a ſimilar polygon GHIK in the baſe B, which is alſo the baſe of a cylinder of the ſame hight.

Demonſtr. The priſms of A and B are in the ſame proportion as their polygon baſes, (*by coroll.* 4. *of the* 39. 11) and the polygons are in the ſame proportion as the circles, (*by coroll.* 2. *of the* 2.) therefore the priſm A has the ſame proportion to the priſm B, as the circle A to the circle B. But as the circle A to the circle B, ſo is the quantity L to the cylinder B: therefore as the priſm A to the priſm B, ſo is the quantity L to the cylinder B. But the priſm A is greater than the quantity L, and conſequently [*by the* 14. 5.] the priſm B, inſcrib'd in the cylinder B, would be greater than its cylinder, which is impoſſible. Therefore neither of the cylinders has a greater proportion to the other, than its baſe to the other's baſe.

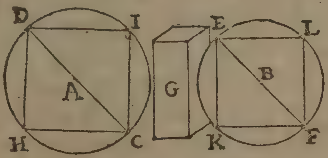
Coroll. Cylinders are triple the cones, of the ſame hight, therefore cones of the ſame hight are in the ſame proportion as their baſes.

PRO

PROPOSITION XII.

A THEOREM.

Cylinders and Cones, that are ſimilar, are in the triplicate proportion of that of the diameters of their baſes.



LET two cones or two cylinders, that are ſimilar, be propos'd, having the circles A and B for their baſes ; I ſay, that the proportion of the cylinder A to the cylinder B is the triplicate proportion of that of the diameter DC to the diameter EF. For if it be not the triplicate proportion, let the quantity G, leſs than the cylinder A, be, to the cylinder B, in the triplicate proportion of that of the diameter DC to the diameter EF; and inſcribe a priſm in the cylinder A greater than G,and another ſimilar to it in the cylinder B : they will be of the ſame hight with the cylinder, becauſe ſimiliar cylinders have their hights and the diameters of their baſes proportional, as well as priſms, (*by defin.* 22. 11.)

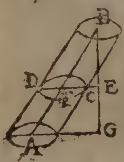
Demonſtr. The diameter DC has the ſame proportion to the diameter EF as the ſide DI to the ſide EL, or as DC to EF, *as I have ſhewn*

in the first. But similar prisms are in the triplicate proportion of that of their homologous sides, (*by coroll.4.of* 39. 11.) therefore the prism A to the prism B is in the triplicate proportion of that of DC to EF. But we suppos'd that the quaintity G in respect of the cylinder B was in the triplicate proportion of that of DC to EF; therefore the prism A will have the same proportion to the prism B as the quantity G to the cylinder B; and(*by the* 14.5.)the prism A being greater than the quantity G, the prism B, inscrib'd in the cylinder B, will be greater than the cylinder B, which is impossible. Therefore similar cylinders are in the triplicate proportion of that of the diameters of their bases.

Cones are the third parts of Cylinders, [*by the* 10.] therefore similar cones are in the triplicate proportion of that of the diameters of their bases.

PROPOSITION XIII.
A THEOREM.

If a cylinder be cut by a plane, that is parellel to its base, the parts of its axis will be in the same proportion as the parts of the cylinder.



LET the cylinder AB be cut by the plane DC parallel to its base: I say, the cylinder AF will have the same proportion to the cylinder FB, as the line AF to the line

line FB. Draw the line BG perpendicular to the plane of the baſe A. Draw alſo upon the planes of the circles DC and A the lines FE and AG. *Demonſtration.*

The plane of the triangle BAG cuts the parallel planes A and DC; therefore the ſections FE and AG are parallel,(*by the* 16.11.) So that AF has the ſame proportion to FB, as the hight GE to EB. Take any aliquot part of EB; and having divided GE and EB into parts equal to it, draw ſo many planes parallel to the baſe A; then will you have ſo many cylinders of the ſame hight; which, having their baſes and hights equal, will be equal, (*by the* 11.)

Further, the lines AF and FB will be divided after the ſame manner as EG and EB, [*by the* 17.11] ſo that the line AF will as oft contain any aliquot part of the line FB, as the cylinder AF contains the like aliquot part of the cylinder FB; therefore the parts of the cylinder will be in the ſame proportion as the parts of their axis.

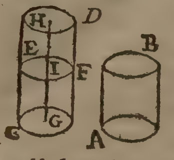
Coroll. The parts of the perpendicular are in the ſame proportion as the parts of the cylinder.

PROP.

PROPOSITION XIV.

A Theorem.

Cylinders and cones, having the same bases, are in the same proportion as their hights.



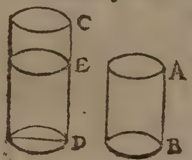
TWO cylinders of equal bases being propos'd, as AB and CD, cut in the greater a cylinder of the same hight with the less, drawing a plane EF parallel to its base. Tis evident [*by the* 11] that the cylinders CF and AB are equal; and that CF has the same proportion to CD, as GI to GH, or [*by the Coroll. of the preceding*] as the hight of CF to the hight of CD; therefore AB has the same proportion to CD, as the hight of CF or AB to the hight of GD.

Cones, being the third parts of cylinders, if their bases be equal, will be also in the same proportion as their hights.

PROPOSITION XV.

A Theorem.

Cylinders and cones that are equal, have their bases and hights reciprocal: and those, that have their bases and hights reciprocal, are equal.



IF the cylinders AB and CD be equal, the base B will have the same proportion to the base D, as the hight CD to

to the hight AB. Take the hight DE equal to the hight AB.

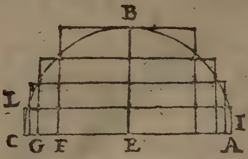
Demonſtr. The cylinder AB has the ſame proportion to the cylinder DE, of the ſame hight, as the baſe B to the baſe D, [*by the* 11.] But as the cylinder AB is to the cylinder DE, ſo is the cylinder CD, equal to AB, to the cylinder DE; *i. e.* ſo is the hight CD to the hight AB or DE. Therefore as the baſe B to the baſe D, ſo is the hight CD to the hight AB.

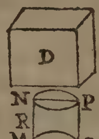
Secondly, if the baſe B has the ſame proportion to the baſe D, as the hight CD to the hight AB, the cylinders AB and CD will be equal. For the cylinder AB is in the ſame proportion to the cylinder DE, as the baſe B to the baſe D: and the cylinder CD will have the ſame proportion to DE, as the hight CD to the hight DE: therefore AB has the ſame proportion to DE, as CD to DE; and [*by the* 9.5.] the cylinders AB and CD will be equal.

"The 16. and 17. Propoſitions are very dif-
"ficult, and of no other uſe but to prove the
"18. which may more eaſily be done by the
"following Lemma's.

LEMMA I.

If a quantity be propos'd leſs than a ſphere, cylinders of the ſame hight may be inſcrib'd in the ſame ſphere greater than that quantity.





‘ Suppoſe A
‘ BC to be
‘ a great ſemi-
‘ circle of the
‘ ſphere, where-
‘ of we treat,
‘and the quantity D to be the quantity leſs
‘than that ſphere: I ſay, ſeveral cylinders of
‘the ſame hight may be inſcrib'd in the ſphere
‘which taken together will be greater than the
‘quantity D. For if the ſemi-ſphere exceed the
‘quantity D, it will exceed it by ſome magni-
‘tude; let it then be the cylinder MP, ſo that
‘the quantities D and MP taken together may
‘be equal to the ſemi-ſphere. Make a great cir-
‘cle of the ſphere to have the ſame proportion
‘to the baſe MO, as the hight MN to the hight
‘R. Then divide the line EB into as many e-
‘qual parts as you pleaſe, each being leſs than
‘R: and drawing parallels to the line AG, de-
‘ſcribe the inſcrib'd and circumſcrib'd paralle-
‘lograms. The number of the circumſcrib'd
‘will exceed that of the inſcrib'd by one. But all
‘the rectangles circumſcrib'd will ſurpaſs all the
‘inſcrib'd by the little rectangles through
‘which the circumference of the circle paſſes:
‘all which taken together are equal to the rect-
‘angle AL. I imagine then the ſemicircle to
‘be turn'd about upon the diameter EB; the ſe-
‘micircle will by that motion deſcribe a ſemi-
‘ſphere,

'ſphere, and the rectangles inſcrib'd ſo many 'cylinders inſcrib'd in the ſemi-ſphere; and the 'circumſcrib'd, other cylinders circumſcrib'd.

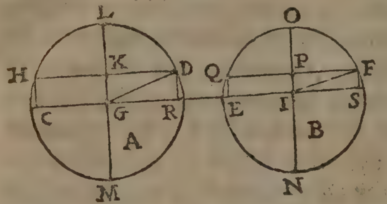
Demonſtr. The circumſcrib'd cylinders ſur-'paſs the inſcrib'd more than the ſemi-ſphere 'ſurpaſſes the ſame inſcrib'd cylinders, it being 'contain'd within the circumſcrib'd cylinders. 'But the circumſcrib'd ſurpaſs the inſcrib'd by 'the cylinder AL: therefore the ſemi-ſphere 'will ſurpaſs thoſe inſcrib'd cylinders by leſs 'than the cylinder deſcrib'd by the rectangle 'AL. But the cylinder AL is leſs than the cy-'linder MP: for there is the ſame proportion 'of a great circle of the ſphere, which is the 'baſe of the cylinder AL, to MO, as of MN to 'R; therefore (*by the preceding*) a cylinder, 'which ſhould have a great circle of the ſphere 'for its baſe, and the hight R, would be equal 'to the cylinder MP: but the cylinder AL, tho 'it have the ſame baſe, yet its hight CL is leſs 'than R; therefore the cylinder AL is leſs than 'the cylinder MP. Conſequently the ſemi-'ſphere, that exceeds the quantity D by the cy-'linder MP, and the inſcrib'd cylinders by a 'quantity leſs than AL, exceeds the inſcrib'd 'cylinders by leſs, than it exceeds the quantity 'D; therefore the quantity D is leſs than the 'cylinders inſcrib'd in the ſemi-ſphere.

'That which I have ſaid of the ſemi-ſphere, is 'applicable to an entire ſphere.

LEM-

LEMMA II.

Similar cylinders, inſcrib'd in two ſpheres, are in the triplicate proportion of the diameters of the ſpheres.



IF two ſimilar cylinders CD and EF be inſcrib'd in the ſpheres A and B, they will be in the triplicate proportion of the diameters LM and NO. Draw the lines GD and IF. *Demonſtration.*

'The right cylinders CD and EF are ſimilar; therefore HD has the ſame proportion to DR as QF to FS, as alſo KD has the ſame proportion to DG as PF to FI. Conſequently the triangles GDK and IFP are ſimilar, (*by the* 6. 6.) therefore KD has the ſame proportion to PF as GD to IF, or LM to ON. But the ſimilar cylinders CD and EF are in the triplicate proportion of KD and PF, the ſemidiameters of their baſes, (*by the* 12.) therefore the ſimilar cylinders CD and EF, inſcrib'd in the ſpheres A and B, are in the triplicate proportion of the diameters of the ſpheres.

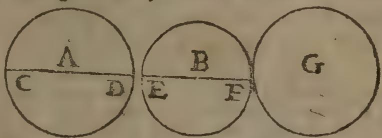
PRO.

PROPOSITION XVIII.

A THEOREM.

Spheres are in the triplicate proportion of their diameters.

THE ſpheres A and B are in the triplicate proportion of that of their diameters CD and EF. For if not, one of the ſpheres, ſuppoſe A, will be in a greater proportion to B, than the triplicate of that of the diameters CD and EF; therefore the quantity G, leſs than the ſphere A, will be in the triplicate proportion of that of CD to EF, to the ſphere B; and then ſome cylinders may (*according to the Lem.* 1) be inſcrib'd in the ſphere A, greater than the quantity G. Inſcribe an equal number of cylinders in the ſphere B, ſimilar to thoſe in the ſphere A.



Demonſtr. The cylinders of the ſphere A to thoſe of the ſphere B are in the triplicate proportion of that of CD to EF, but the quantity G to the ſphere B is alſo in the triplicate proportion of that of CD to EF: therefore the cylinders of the ſphere A have the ſame proportion to the ſimilar cylinders of the ſphere B, as the

the quantity G to the ſphere B. Conſequently the cylinders of A being greater than the quantity G, the cylinders of B, *i.e.* inſcrib'd in the ſphere B, will be greater than the ſphere B, which is impoſſible. Therefore the ſpheres A and B are in the triplicate proportion of that of their diameters.

Coroll. Spheres are in the ſame proportion as the cubes of their diameters; becauſe cubes, being ſimilar ſolids, are in the triplicate proportion of their ſides, [*by the* 33. 11.]

FINIS.
